Prof. Dr. Andreas Podelski
Dr. Matthias Heizmann
Christian Schilling

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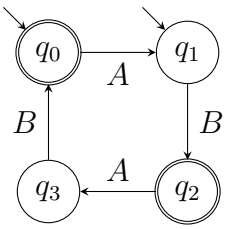
Tutorial for Cyber-Physical Systems - Discrete Models
Exercise Sheet 10

On this exercise sheet you may again use the web interface of the ULTIMATE Automata Library [1]. Wherever you have to provide an automaton, you may alternatively use the notation from the web interface and submit the automata declarations to your tutor via email. The sample file `BuchiAutomata.ats` from the web interface shows you how to declare NBA and how to apply operations. Note that since NFA and NBA are syntactically similar, both are represented via the same datatype, but the names for the operations are different. E.g., the NFA acceptance check is `Accepts`, whereas the NBA acceptance check is `BuchiAccepts`.

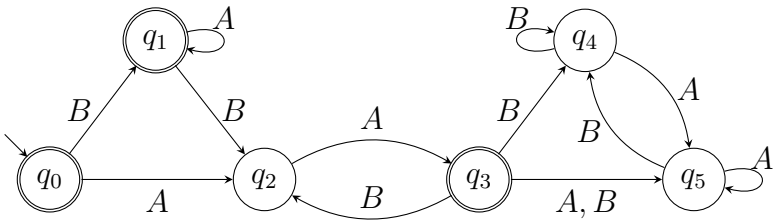
Exercise 1: From NBA to ω-regular expressions 3 Points

Construct for each of the following NBA an ω-regular expression that describes the same language. The alphabet of each automaton is $\Sigma = \{A, B\}$.

(a) Apply the construction from the lecture (slide 42 ff. of lecture 19). You do not have to write down each NFA $\mathcal{A}_{q,p}$ that you construct, but you have to write down the regular expression for each NFA that you construct.



(b) This time you need not apply the construction from the lecture. You may use optimizations like omitting automata whose language is empty.



[1] `https://ultimate.informatik.uni-freiburg.de/automata_script_interpreter`

Exercise 2: From ω-regular expression to NBA 2 Points

Construct an NBA for the following ω-regular expression over the alphabet $\Sigma = \{A, B\}$.

$$\underbrace{(A + BA^*)}_{\alpha_1} . \underbrace{A}_{\beta_1}{}^{\omega} + \underbrace{BB}_{\alpha_2} . \underbrace{(A + AB)}_{\beta_2}{}^{\omega}$$

Apply the construction from the lecture (slide 56 ff. of lecture 19) by executing the following steps for $i = 1, 2$.

(a) Construct an NFA $\mathcal{A}_i$ for each regular expression α_i, resp. an NFA $\mathcal{B}_i$ for each regular expression β_i.

(b) Use the NFA $\mathcal{B}_i$ to construct an NBA $\mathcal{B}_i^\omega$ (for each β_i).

(c) Use the NFA $\mathcal{A}_i$ and the NBA $\mathcal{B}_i^\omega$ to construct an NBA $\mathcal{C}_i$ for each $\alpha_i.\beta_i^\omega$.

(d) Construct the final NBA by taking the union of all NBA $\mathcal{C}_i$.

Exercise 3: From GNBA to NBA 2 Points

Consider the GNBA outlined below with acceptance sets $F_1 = \{q_1\}$ and $F_2 = \{q_2\}$. Construct an NBA that accepts the same language.