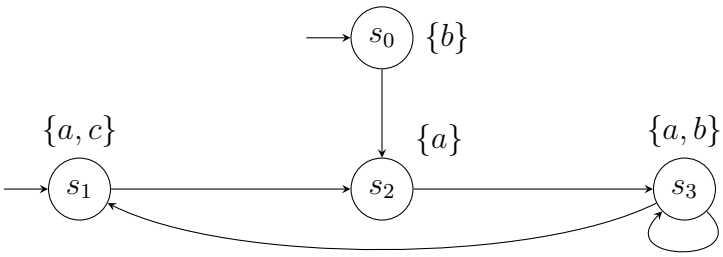
Prof. Dr. Andreas Podelski
Dr. Matthias Heizmann
Christian Schilling

Delivery: February 6th, 2017
16:15 via the post boxes
Discussion: February 8th, 2017

Tutorial for Cyber-Physical Systems - Discrete Models
Exercise Sheet 14

Exercise 1: CTL 2.5+0.5 Points

Consider the following transition system over the set of atomic propositions $\{a, b, c\}$:



Indicate for each of the following CTL formulas the set of states for which the formula is satisfied.

(a) $a \wedge \exists \bigcirc b$
(b) $\forall \bigcirc \exists \bigcirc c$
(c) $\forall(a \ \mathsf{U} \ b)$
(d) $\neg(a \ \mathsf{U} \ \Box b)$
(e) $\forall((\exists \Diamond c) \ \mathsf{U} \ (\forall \Box a))$
(f) $\forall \Box \forall \Diamond b$

Exercise 2: Equivalences CTL and LTL 3+1 Points

(a) Consider the LTL formula $\varphi_1 := \Diamond(a \wedge \bigcirc a)$. Show that the following CTL formulas are not equivalent to φ_1:

(i) $\psi_1 := \forall \Diamond(a \wedge \exists \bigcirc a)$ (ii) $\psi_2 := \exists \Diamond(a \wedge \forall \bigcirc a)$ (iii) $\psi_3 := \exists \Diamond(a \wedge \exists \bigcirc a)$

Bonus: Find two transition systems which show that φ_1, ψ_1, ψ_2, and ψ_3 are pairwise not equivalent.

(b) Consider the LTL formula $\varphi_2 := \Diamond \Box a$. Show that the following CTL formulas are not equivalent to φ_2:

(i) $\psi_4 := \forall \Diamond \exists \Box a$ (ii) $\psi_5 := \exists \Diamond \forall \Box a$ (iii) $\psi_6 := \exists \Diamond \exists \Box a$

Bonus: Find two transition systems which show that φ_2, ψ_4, ψ_5, and ψ_6 are pairwise not equivalent.

Hint: To show that two formulas φ and ψ are not equivalent, depict a transition system TS such that $TS \models \varphi$ and $TS \not\models \psi$ (or the other way around).

Exercise 3: Existential Normal Form 2 Points

Convert the following CTL formulas into existential normal form:

(a) $\forall \bigcirc a$
(b) $\forall \Box \forall \Diamond a$
(c) $\forall(a \ \mathsf{U} \ b)$