



# Robot Map Optimization using Dynamic Covariance Scaling

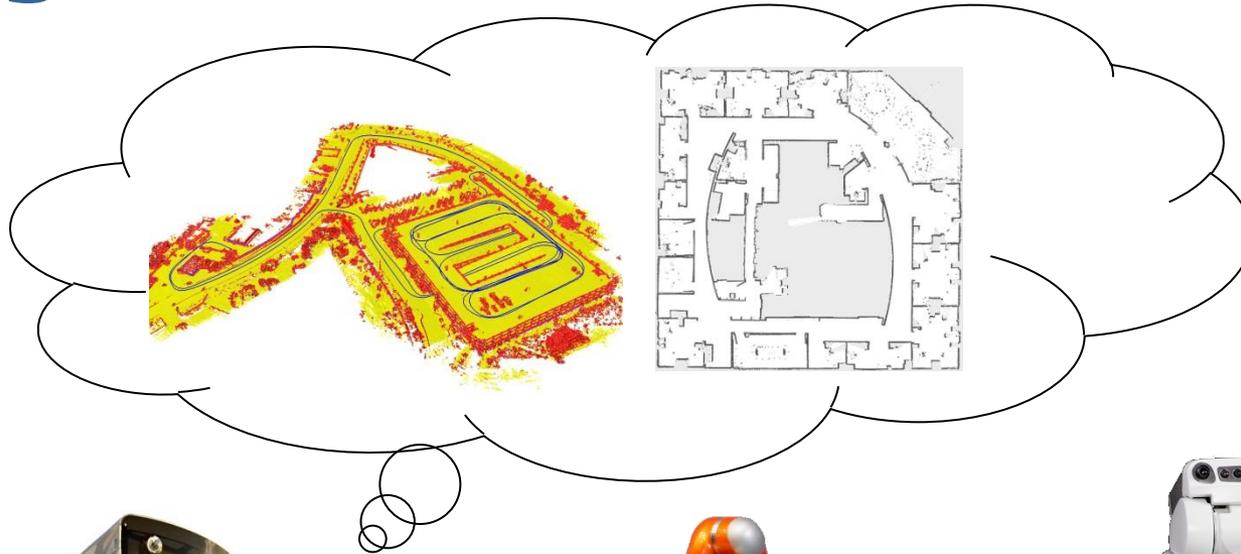
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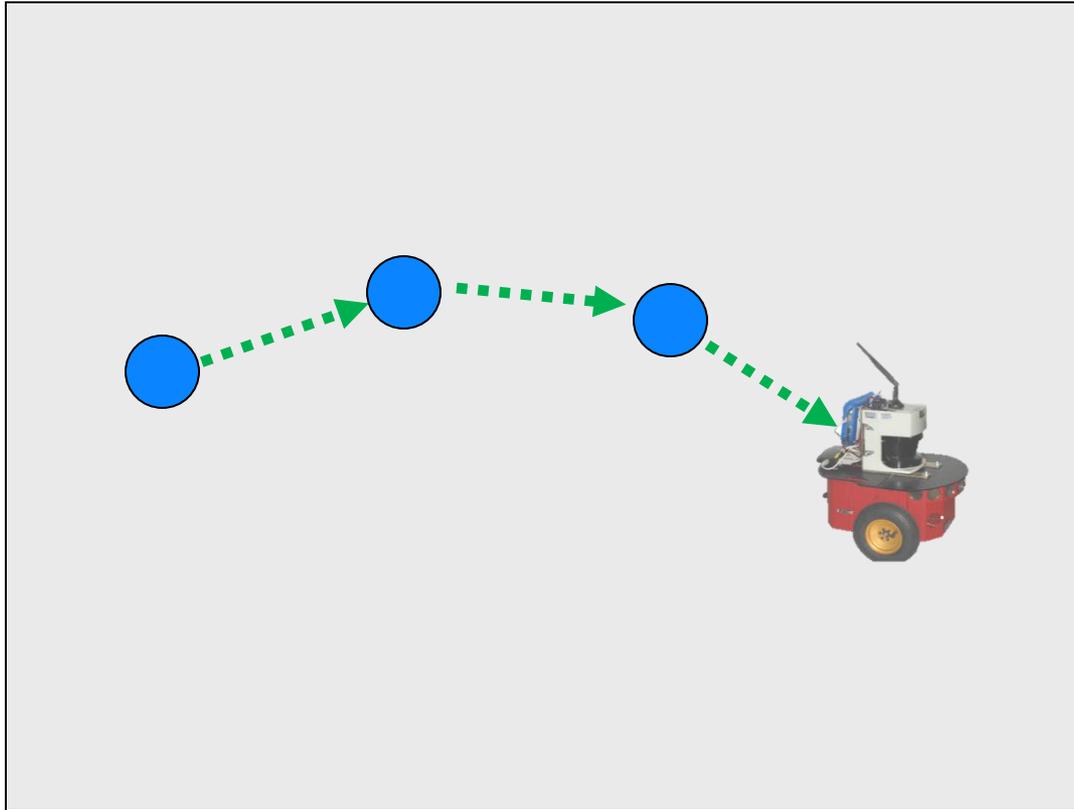
**Pratik Agarwal**, Gian Diego Tipaldi, Luciano Spinello,  
Cyrill Stachniss and Wolfram Burgard

University of Freiburg, Germany

# Maps are Essential for Effective Navigation



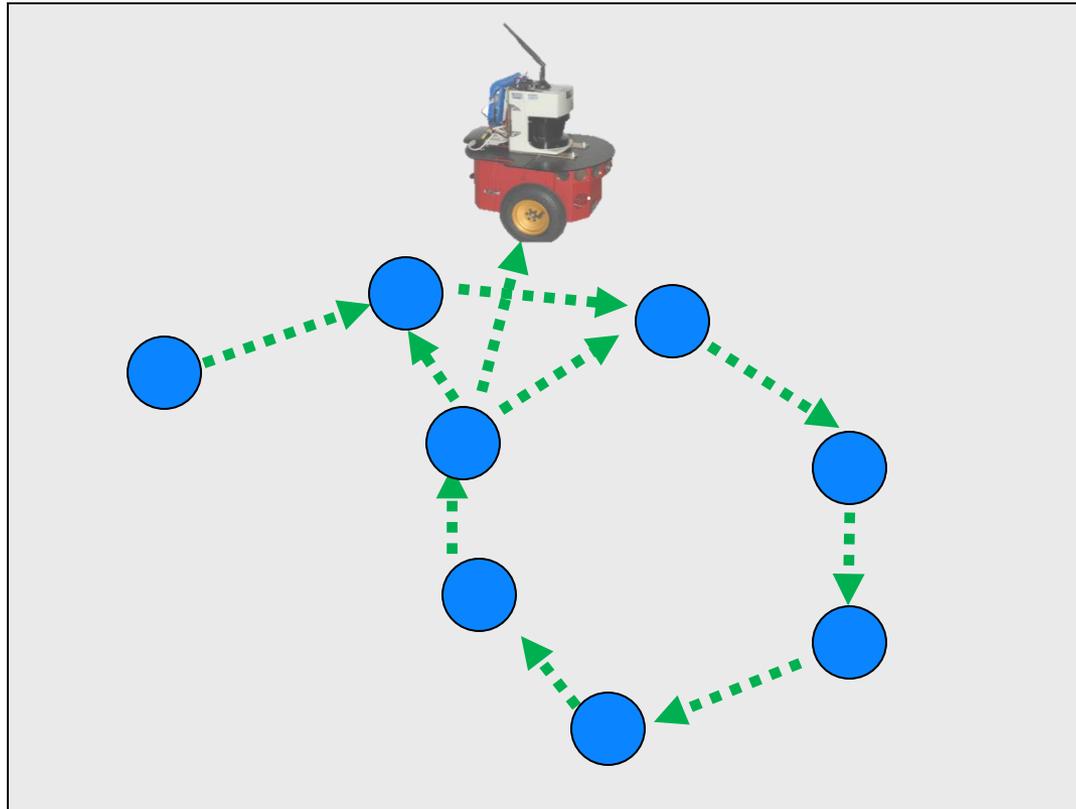
# Graph-based SLAM



● Robot pose

→ Constraint

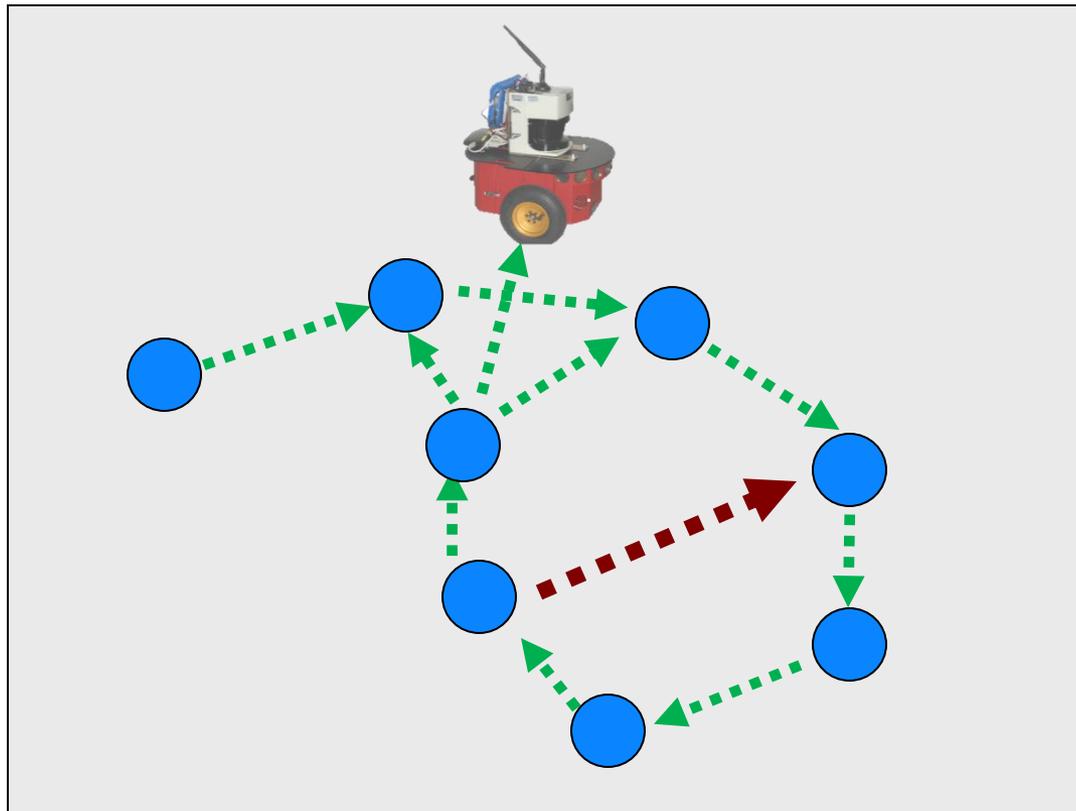
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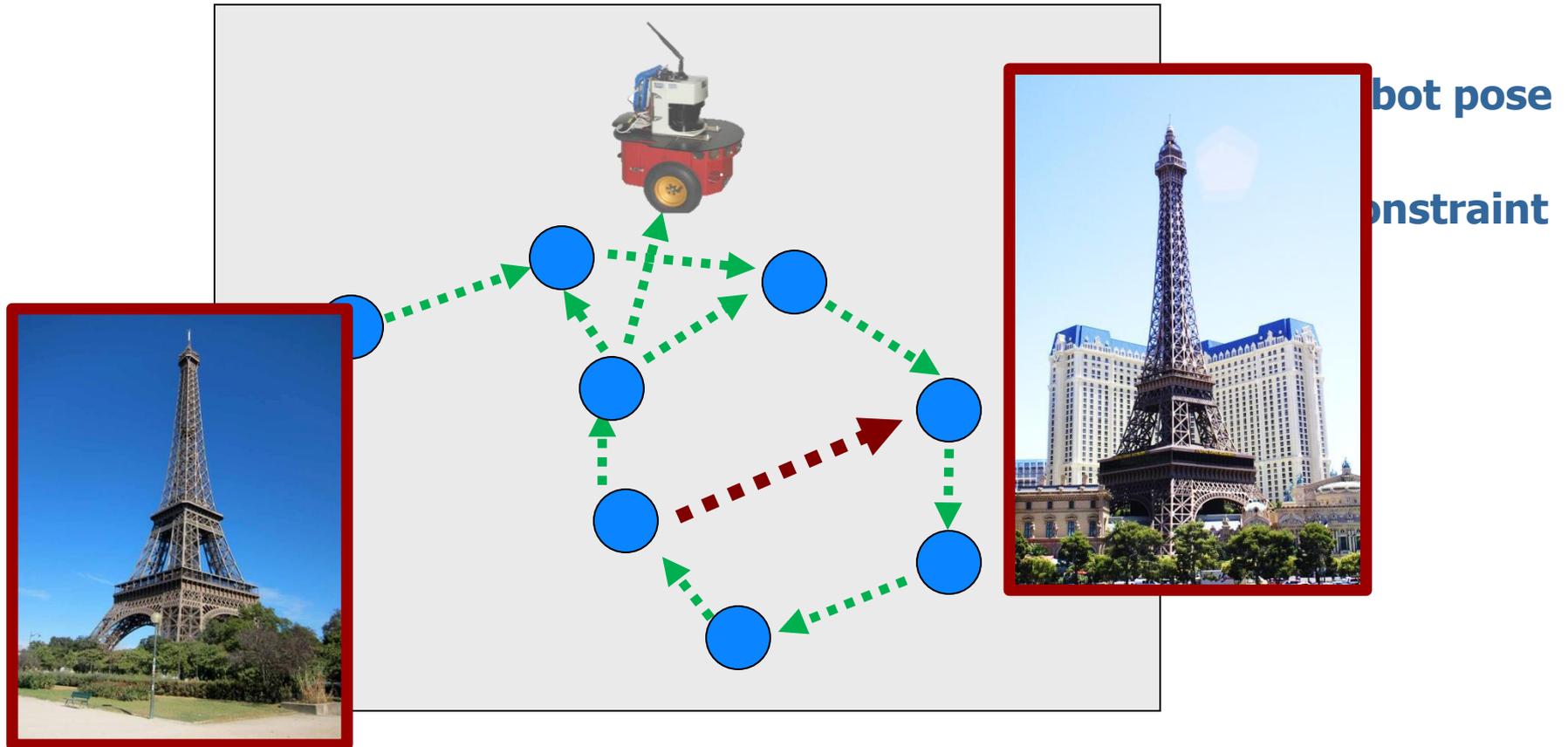


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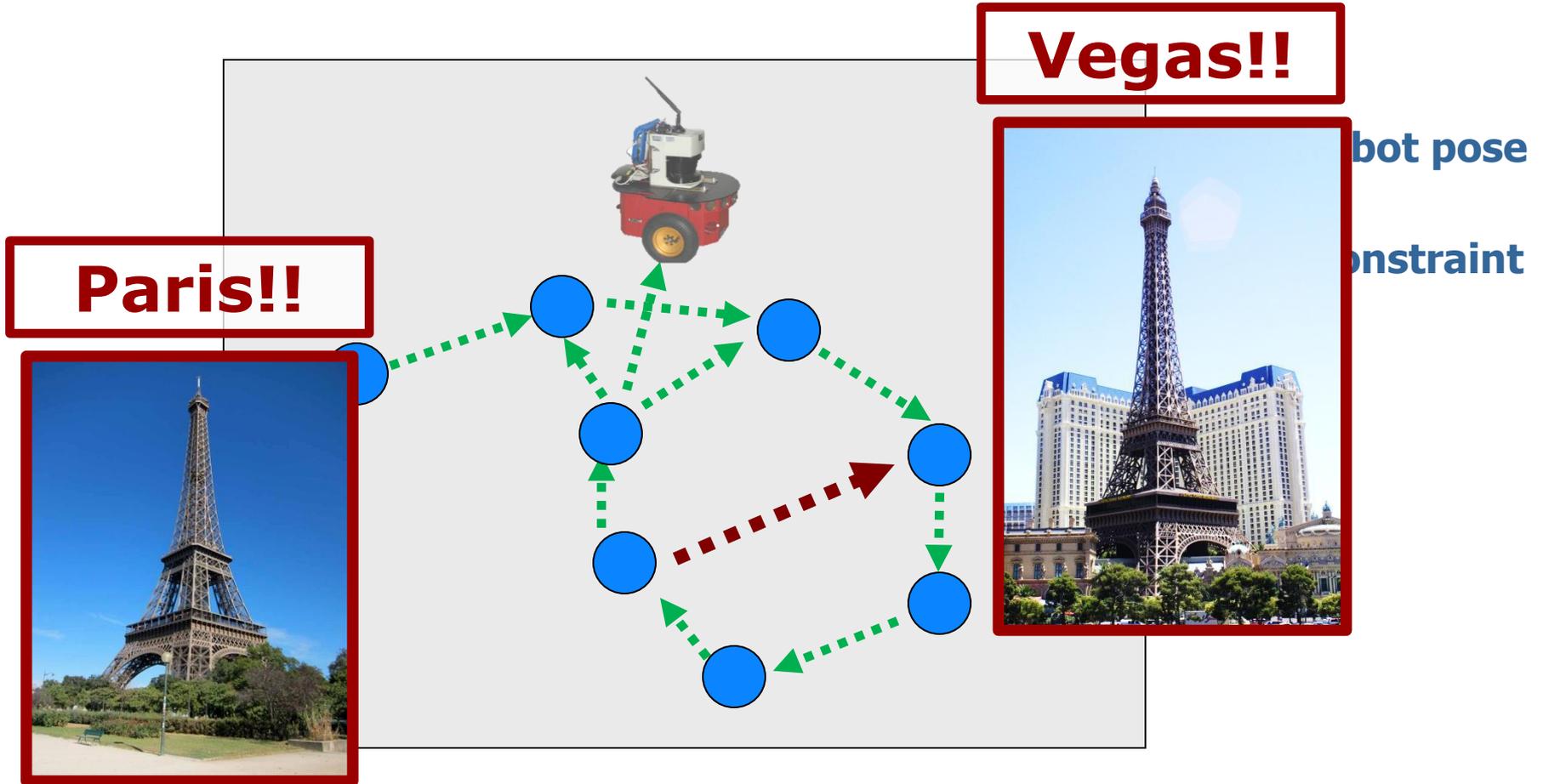
**a single outlier ...**

# Graph-based SLAM



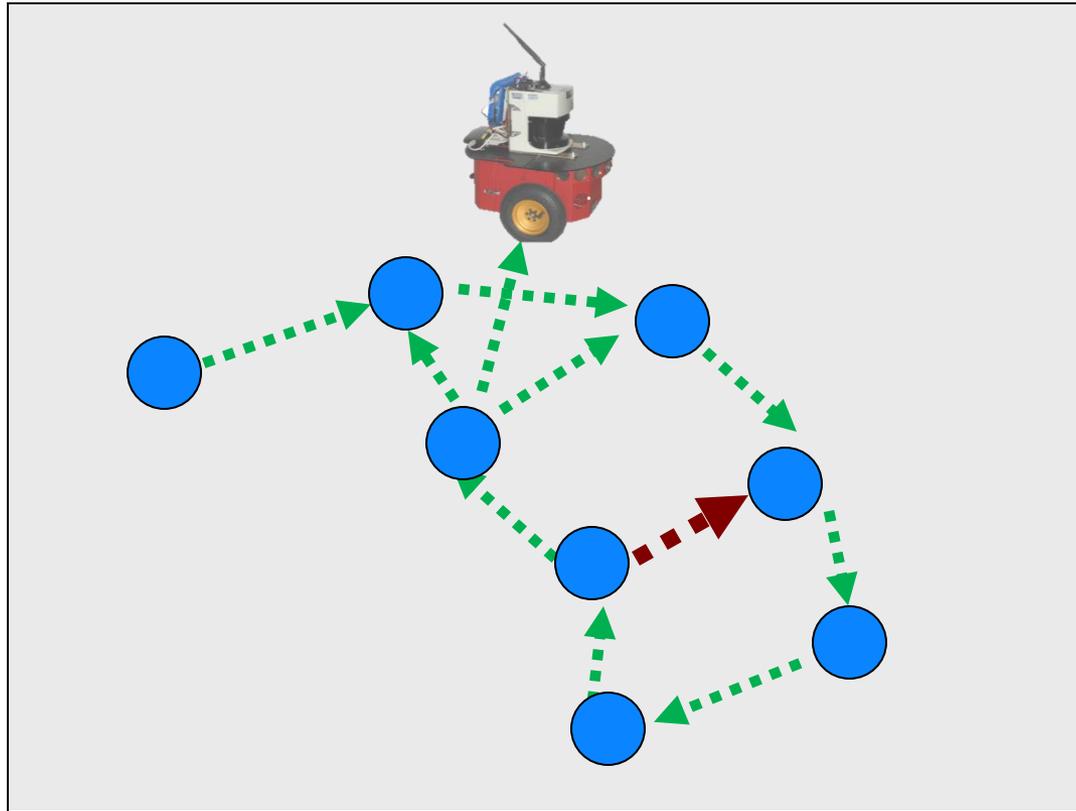
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# Graph-based SLAM



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# Graph-based SLAM



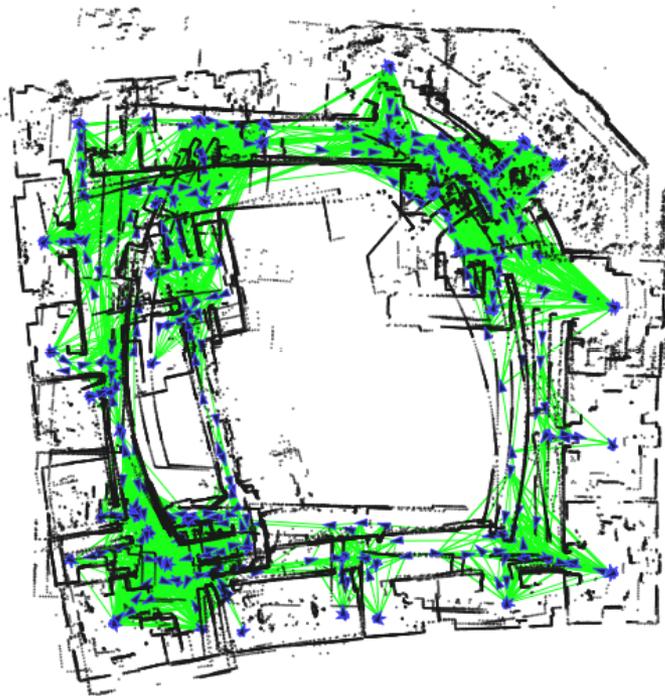
● Robot pose

→ Constraint

**a single outlier ... ruins the map**

# Graph-SLAM Pipeline

Front end  Validation  Back end



**Assumption:**  
  
**No  
Outliers**

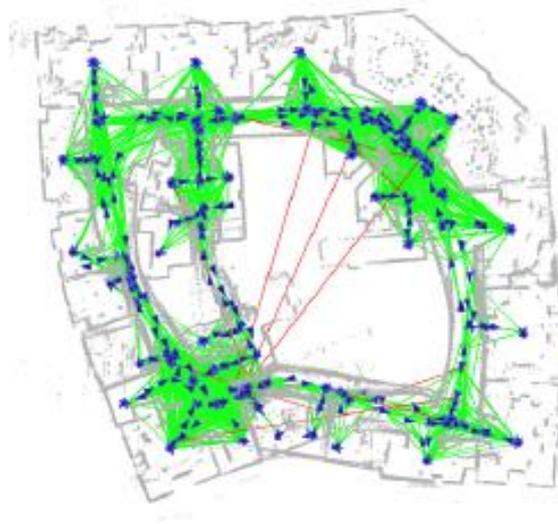


**Impossible to have perfect validation**

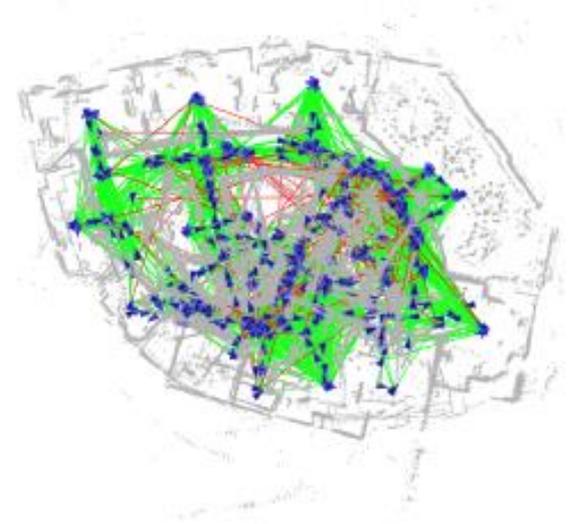
# SLAM Back End Fail in the Presence of Outliers



**1  
Outlier**



**10  
Outliers**



**100  
Outliers**

# Why does the Mapping Fail?

- **Gaussian assumption is violated**
- Perceptual aliasing
- Measurement error
- Multipath GPS measurements

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## Alternative

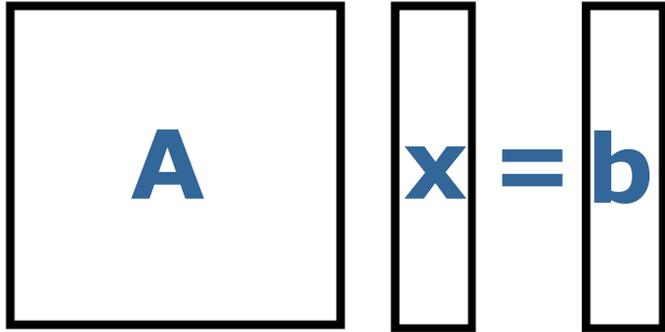
- **Let the back end do the validation job!**

# Switchable Constraints

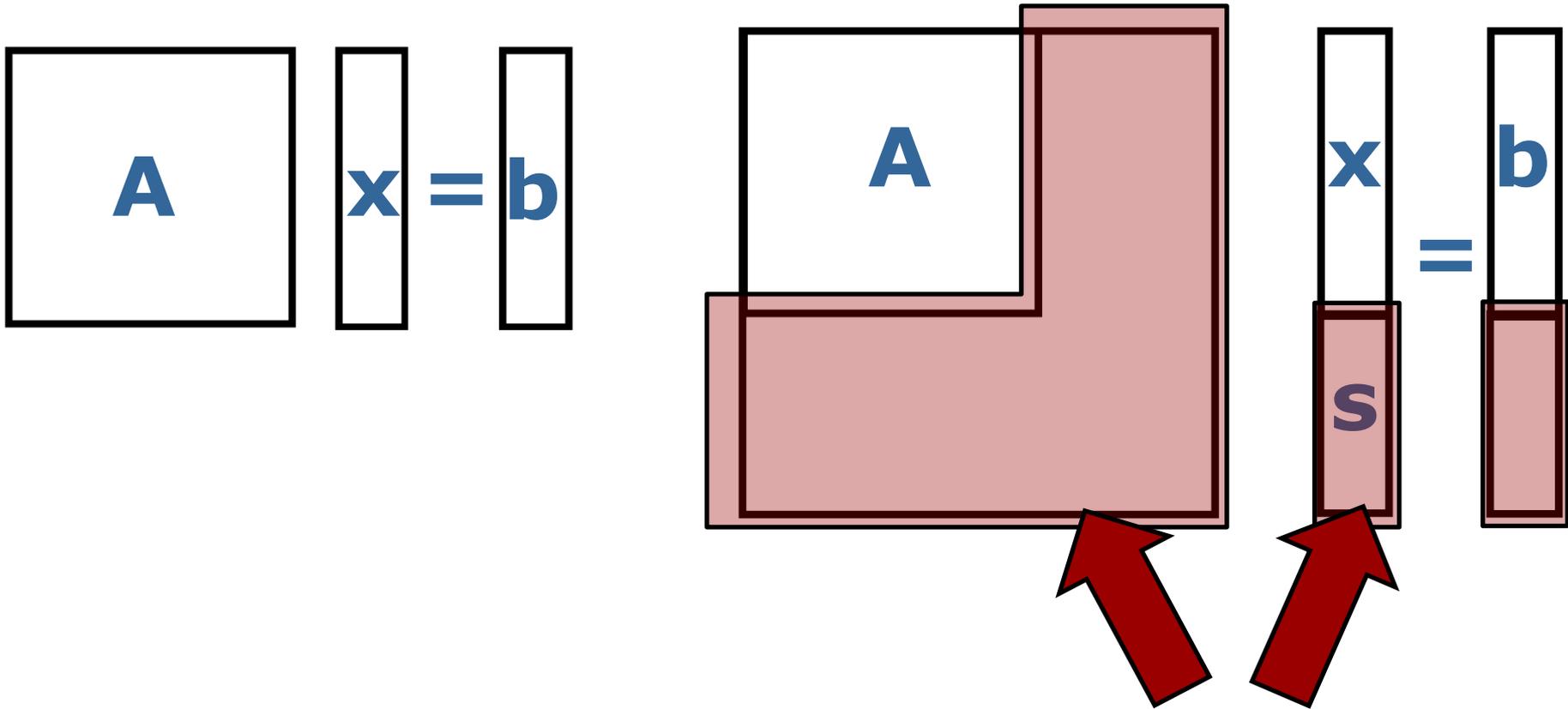
- Recent approach by Sünderhauf and Protzel
- Allows to switch off “bad” constraints
- Requires a **switch-variable** per constraint
- Additional **switch-constraint** per switch
- Robust optimization in case of outliers but increased computational complexity

[Sünderhauf and Protzel, IROS 2012]

# Standard Optimization

$$\mathbf{A} \mathbf{x} = \mathbf{b}$$
The diagram illustrates the linear equation  $\mathbf{A} \mathbf{x} = \mathbf{b}$ . The matrix  $\mathbf{A}$  is represented by a square box, indicating it is a square matrix. The vector  $\mathbf{x}$  is represented by a tall, narrow vertical rectangle, indicating it is a column vector. The vector  $\mathbf{b}$  is also represented by a tall, narrow vertical rectangle, indicating it is a column vector. The equals sign is placed between the vector  $\mathbf{x}$  and the vector  $\mathbf{b}$ .

# Switchable Constraints



- Additional **switch-variable** per constraint
- Additional **switch-constraint** per switch

# Our Approach: Dynamic Covariance Scaling

- Switch variables **computed in closed form**
- Does not increase state space
- Approximates Switchable Constraints
- Is a robust M-estimator
- Successfully **rejects** outliers

# Standard Gaussian Least Squares

$$X^* = \operatorname{argmin}_X \sum_{ij} \underbrace{\mathbf{e}_{ij}(X)^T \Omega_{ij} \mathbf{e}_{ij}(X)}_{\chi_{ij}^2}$$

# Dynamic Covariance Scaling

$$X^* = \operatorname{argmin}_X \sum_{ij} \underbrace{\mathbf{e}_{ij}(X)^T \Omega_{ij} \mathbf{e}_{ij}(X)}_{\chi_{ij}^2}$$

$$X^* = \operatorname{argmin}_X \sum_{ij} \mathbf{e}_{ij}(X)^T (s_{ij}^2 \Omega_{ij}) \mathbf{e}_{ij}(X)$$


# How to Determine $s$ ?

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$$X^*, S^* = \operatorname{argmin}_{X, S} \underbrace{g(X_{ij \neq kl}, S_{ij \neq kl})}_{b} + \underbrace{s_{kl}^2 \chi_{kl}^2 + (1 - s_{kl})^2 \Phi}_{h(s, \chi^2)}$$

$$\nabla b = \begin{bmatrix} \vdots \\ \frac{\partial b}{\partial s} \\ \vdots \end{bmatrix} = \begin{bmatrix} \vdots \\ 2s\chi_l^2 - 2(1-s)\Phi \\ \vdots \end{bmatrix} = \begin{bmatrix} \vdots \\ 0 \\ \vdots \end{bmatrix}$$

$$\hat{h} = \frac{\Phi^2 \chi_l^2}{(\chi_l^2 + \Phi)^2} + \Phi - \frac{2\Phi^2}{\chi_l^2 + \Phi} + \frac{\Phi^3}{(\chi_l^2 + \Phi)^2}$$

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**Check out the paper!**

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$$\rightarrow s_{ij} = \min \left( 1, \frac{2\Phi}{\Phi + \chi_{ij}^2} \right)$$

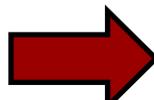
**Closed form**

**Check out the paper!**

# How to Determine $s$ ?

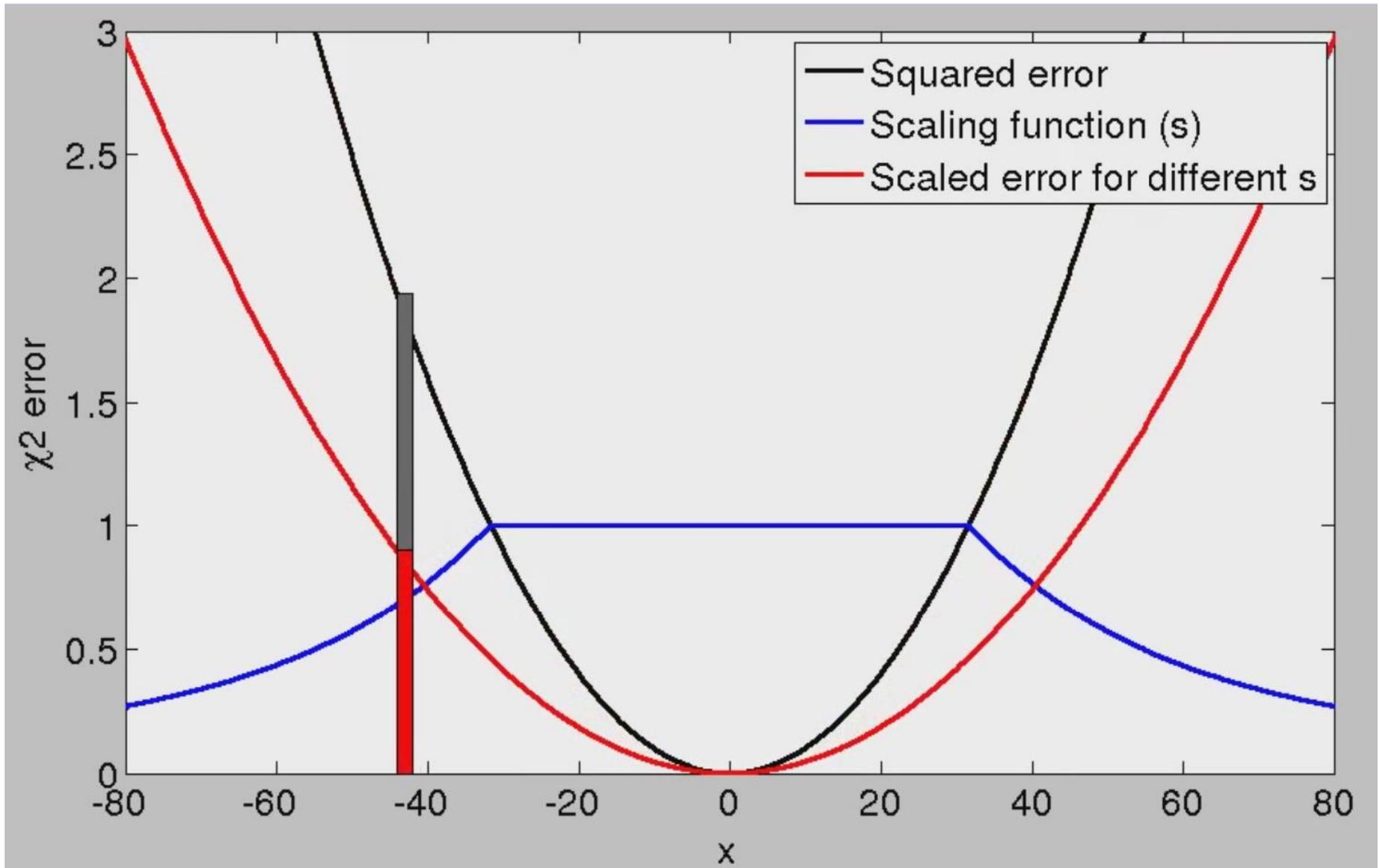
$$X^* = \operatorname{argmin}_X \sum_{ij} \mathbf{e}_{ij}(X)^T \left( s_{ij}^2 \Omega_{ij} \right) \mathbf{e}_{ij}(X)$$


▪  
▪  
▪

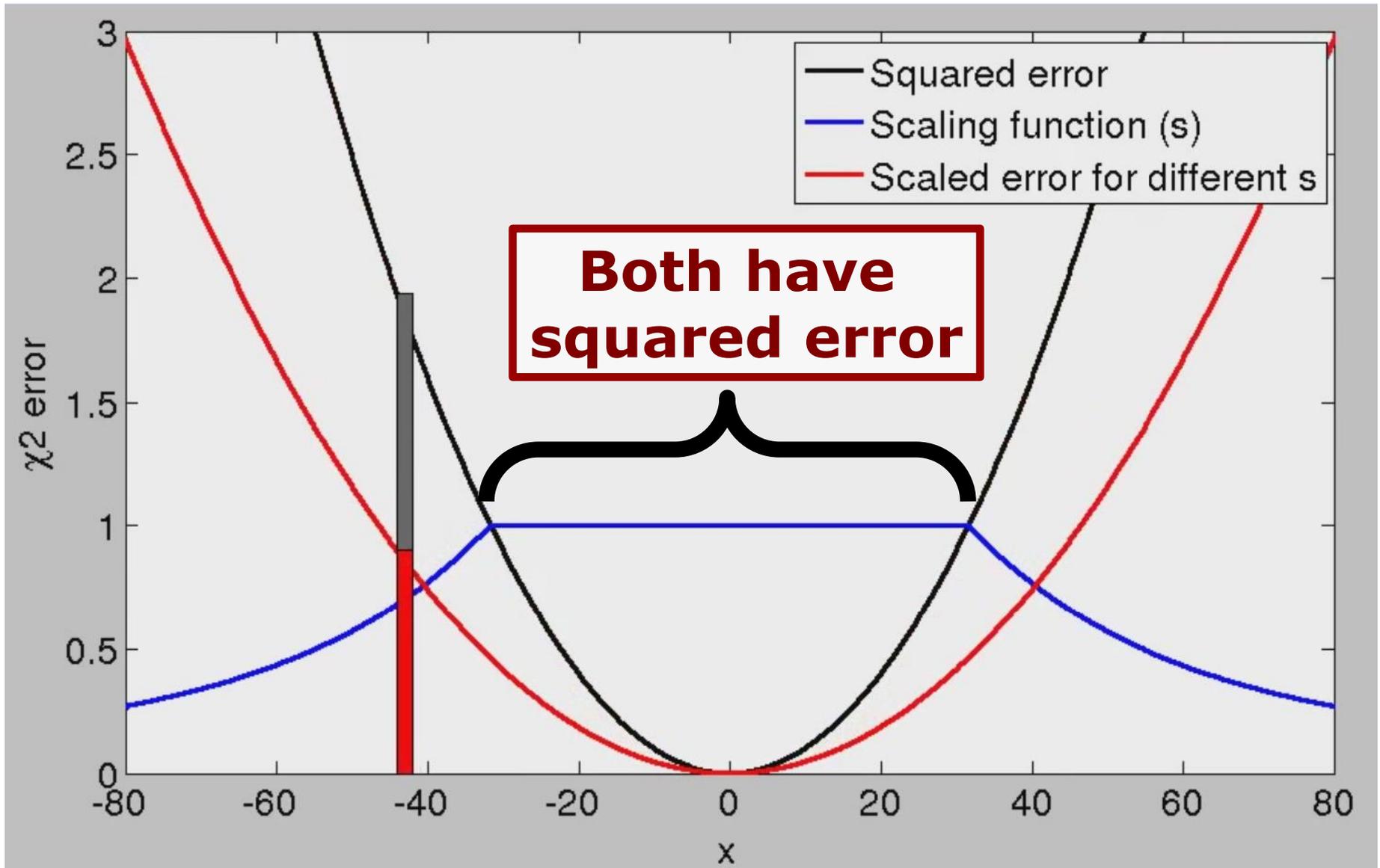

$$s_{ij} = \min \left( 1, \frac{2\Phi}{\Phi + \chi_{ij}^2} \right)$$

**Closed form**

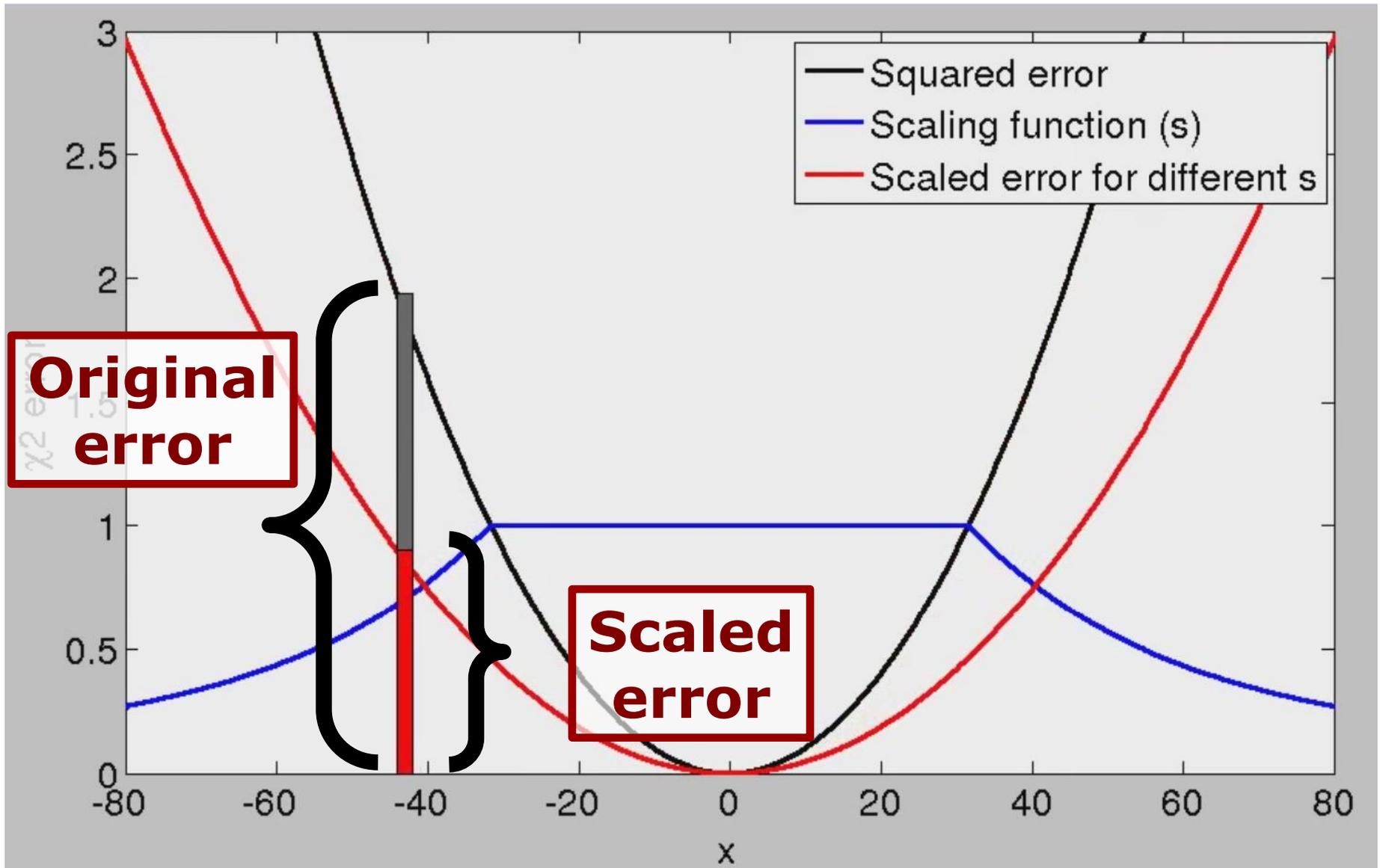
# Dynamic Covariance Scaling



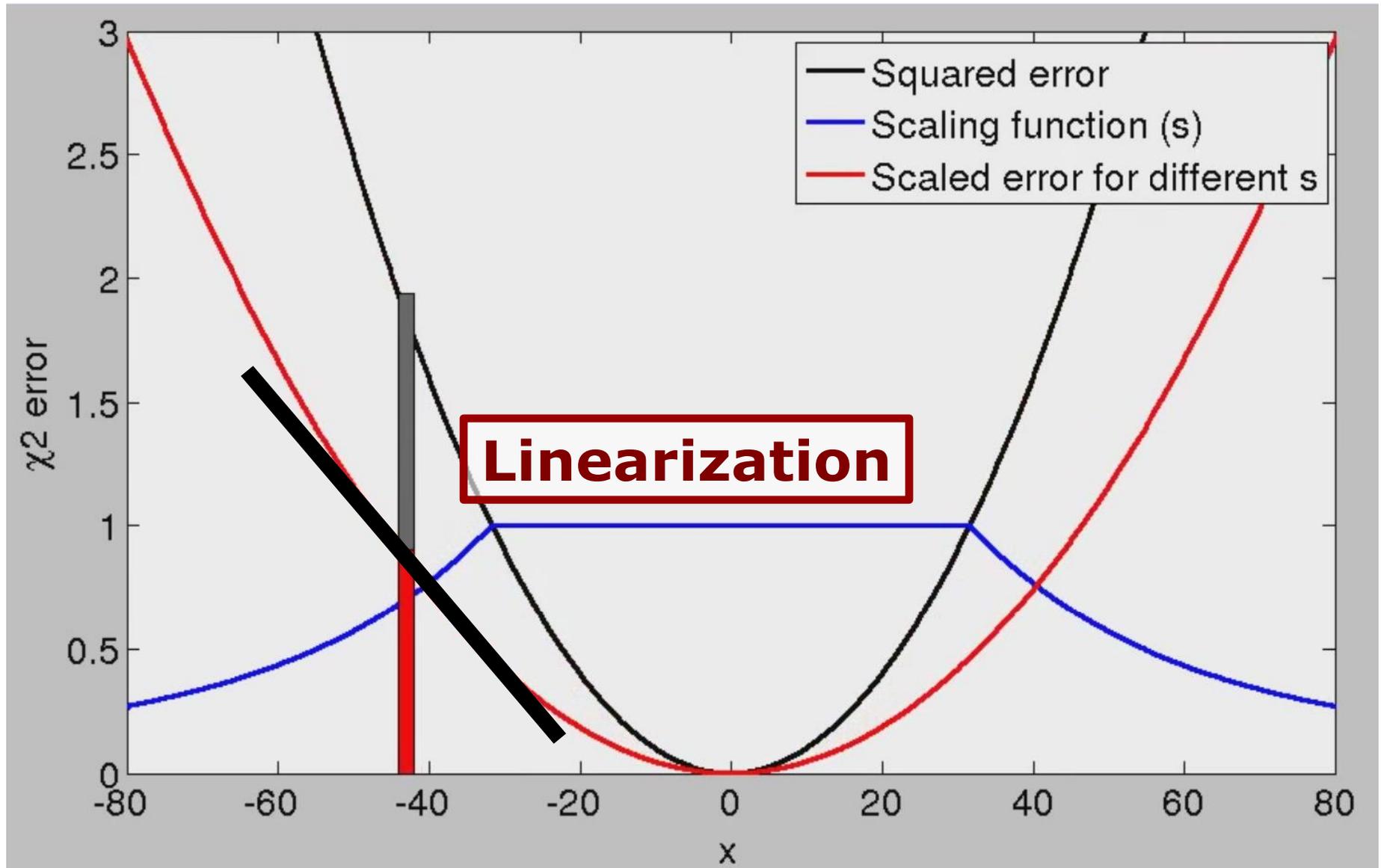
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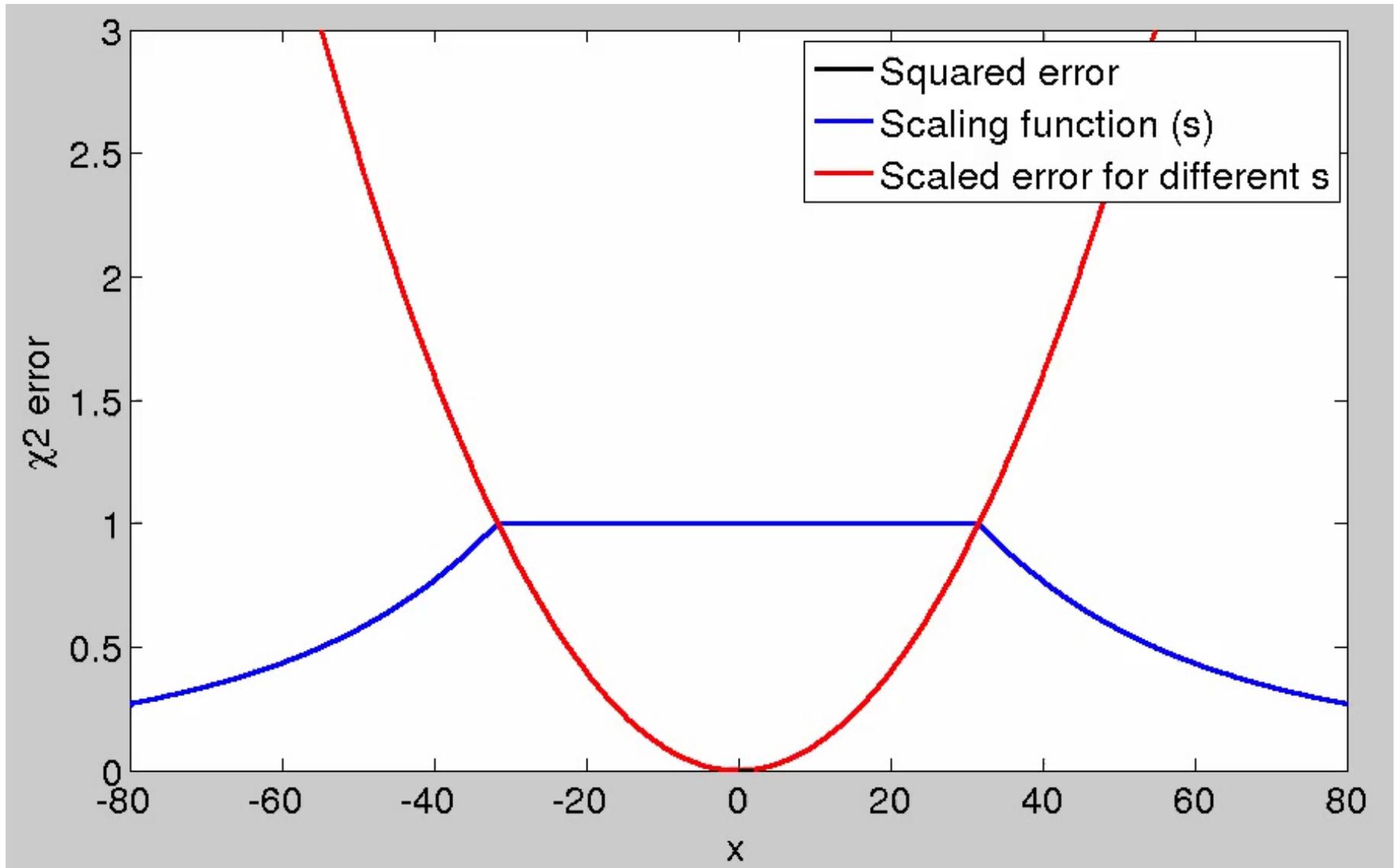
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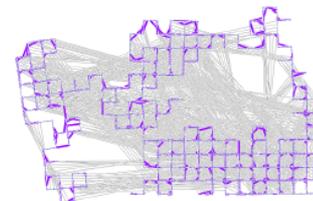
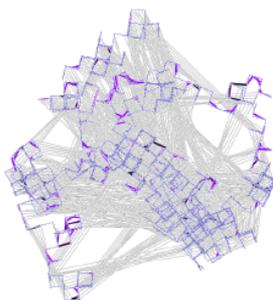
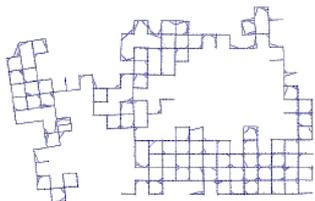
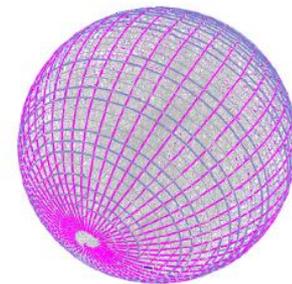
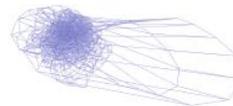
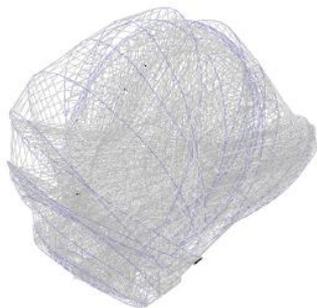
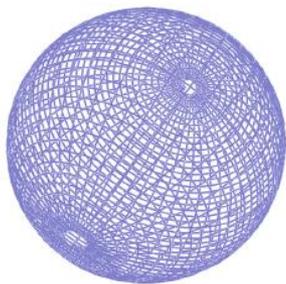
# Robust SLAM with Our Method

Ground  
Truth

Initialization

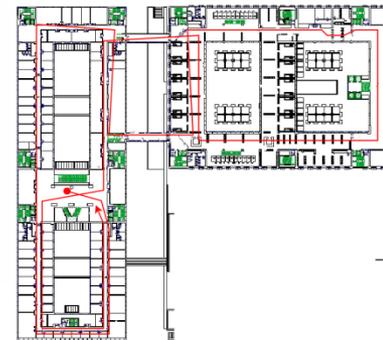
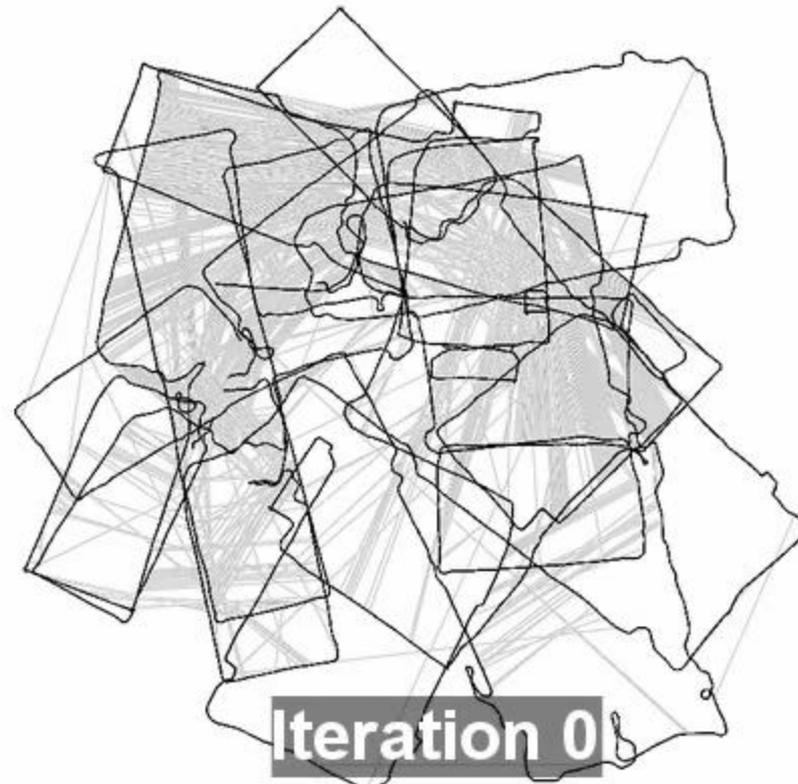
Gauss  
Newton

**Our  
Method**



Manhattan3500 (1000 Outliers)  
Sphere2500 (1000 Outliers)

# Dynamic Covariance Scaling with Front-end Outliers



Bicocca-multisession dataset

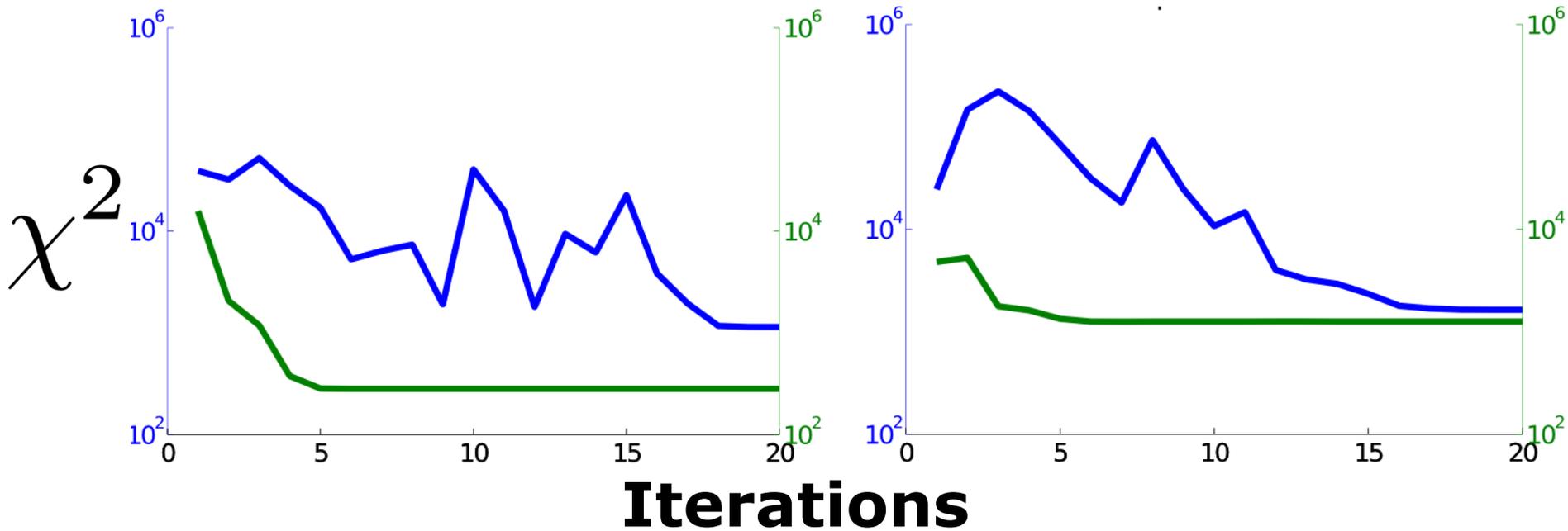
# Convergence – 1000 Outliers

■ **Switchable Constraints**

■ **Dynamic Covariance Scaling**

## Manhattan3500

## Sphere2500



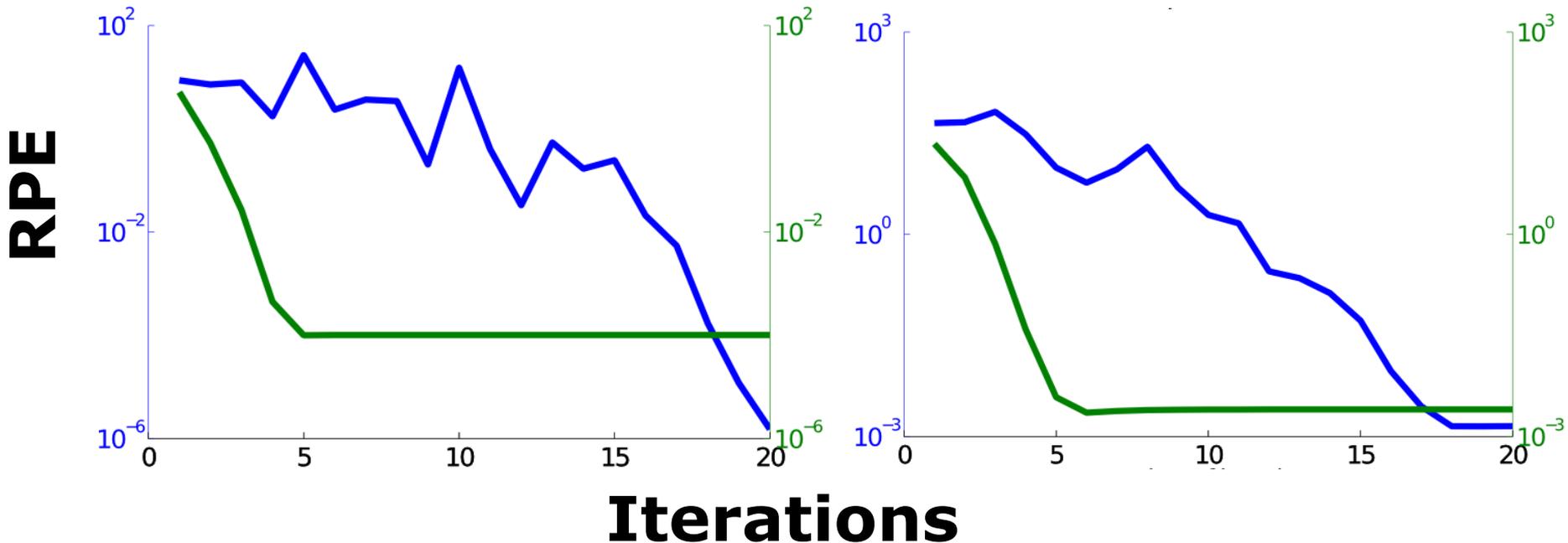
# Convergence – 1000 Outliers

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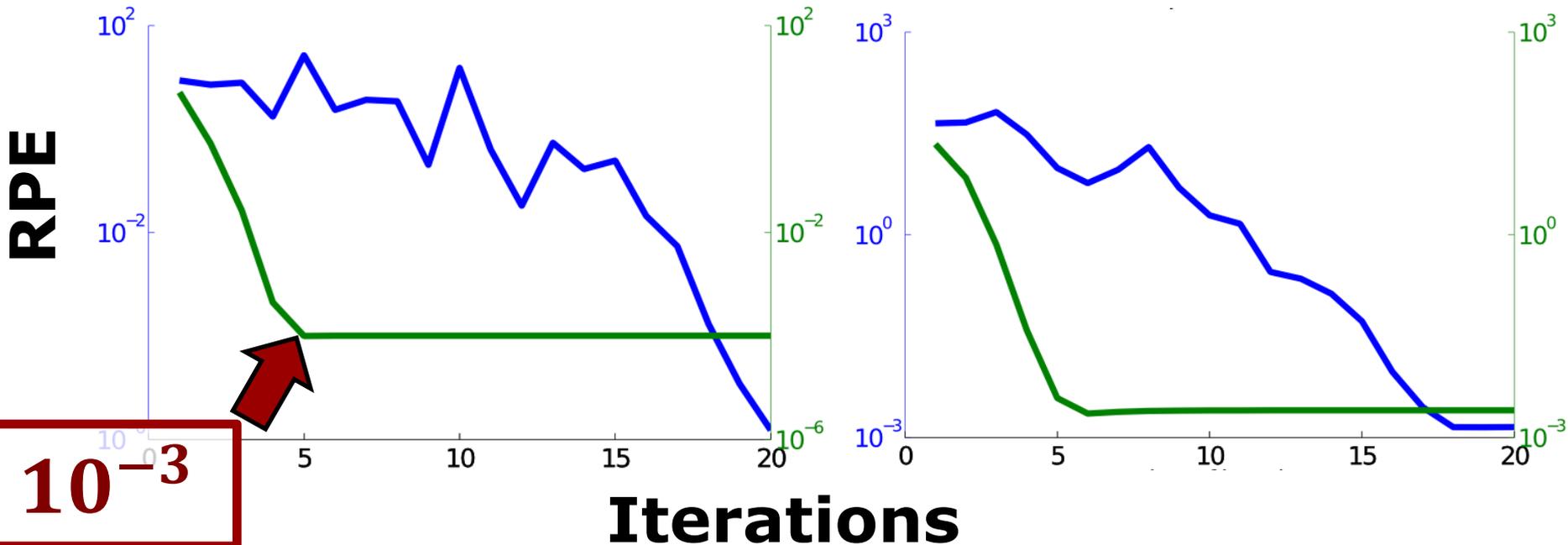


# Convergence – 1000 Outliers

- Switchable Constraints
- Dynamic Covariance Scaling

## Manhattan3500

## Sphere2500



# Convergence with Outliers

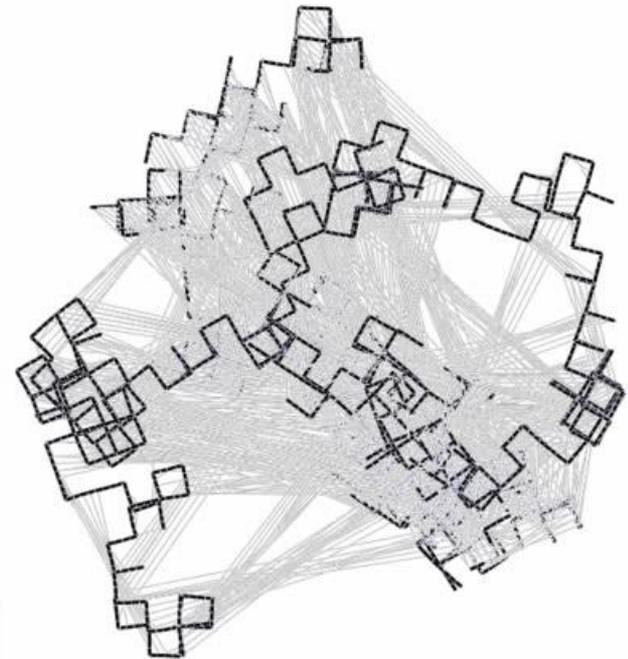
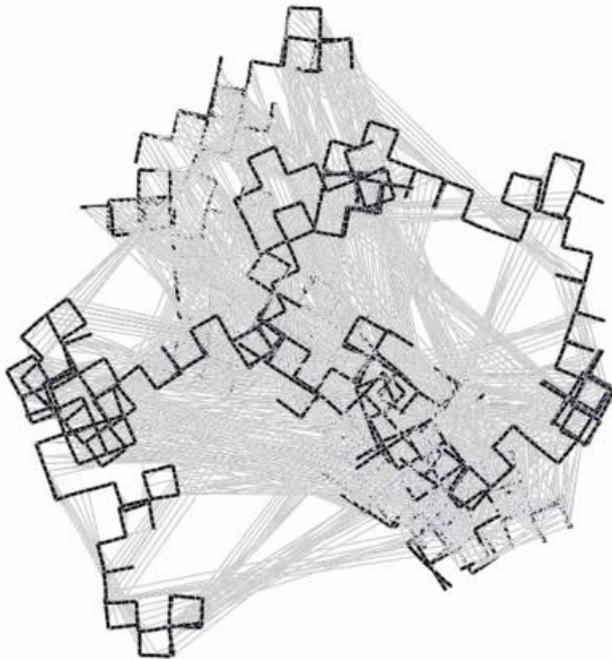
## Switchable Constraints

## Dynamic Covariance Scaling

Switchable Constraints (SC)

ManhattanOlson

Dynamic Covariance Scaling (DCS)



Iteration 0

# Conclusion

- **Rejects outliers** for 2D & 3D SLAM
- **No increase in computational complexity**
- Approximates switchable constraints with a robust M-estimator
- Now **integrated in g2o**

**Thank you for your attention!**

# Questions?

