Helmert's and Bowie's Geodetic Mapping Methods and their Relationship to Graph-Based SLAM

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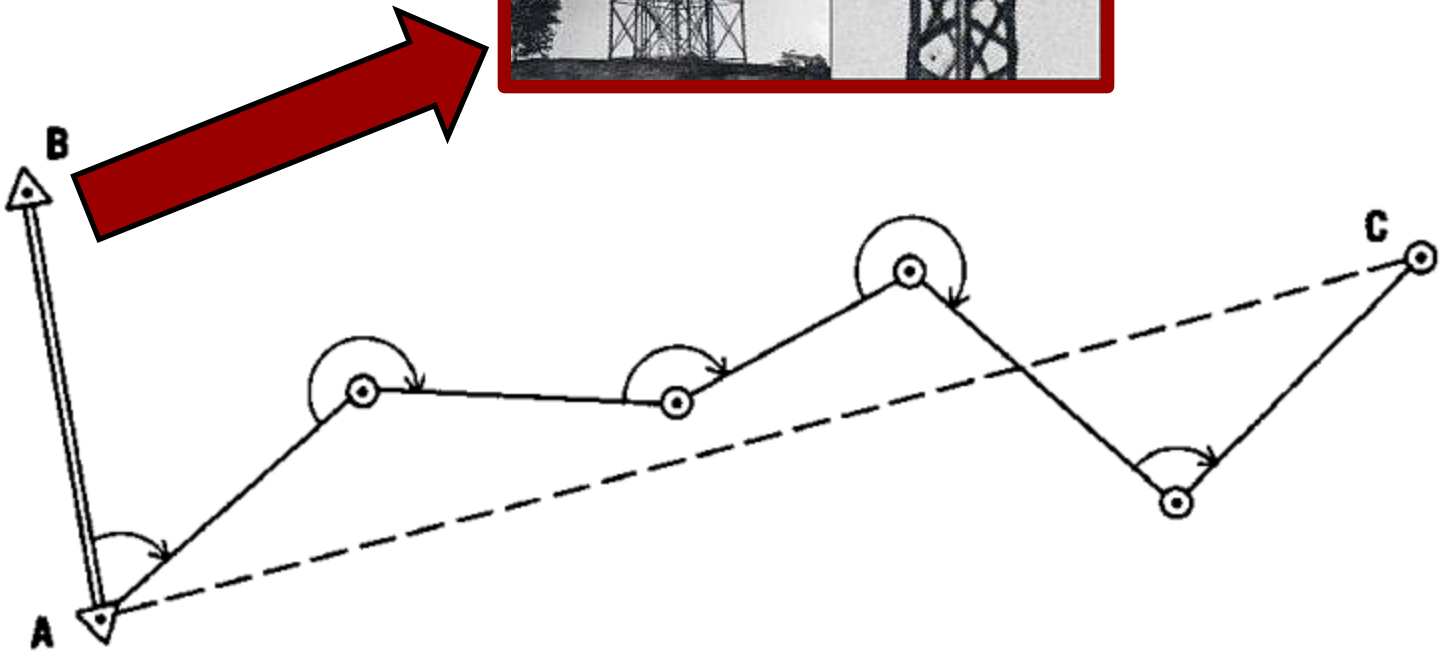
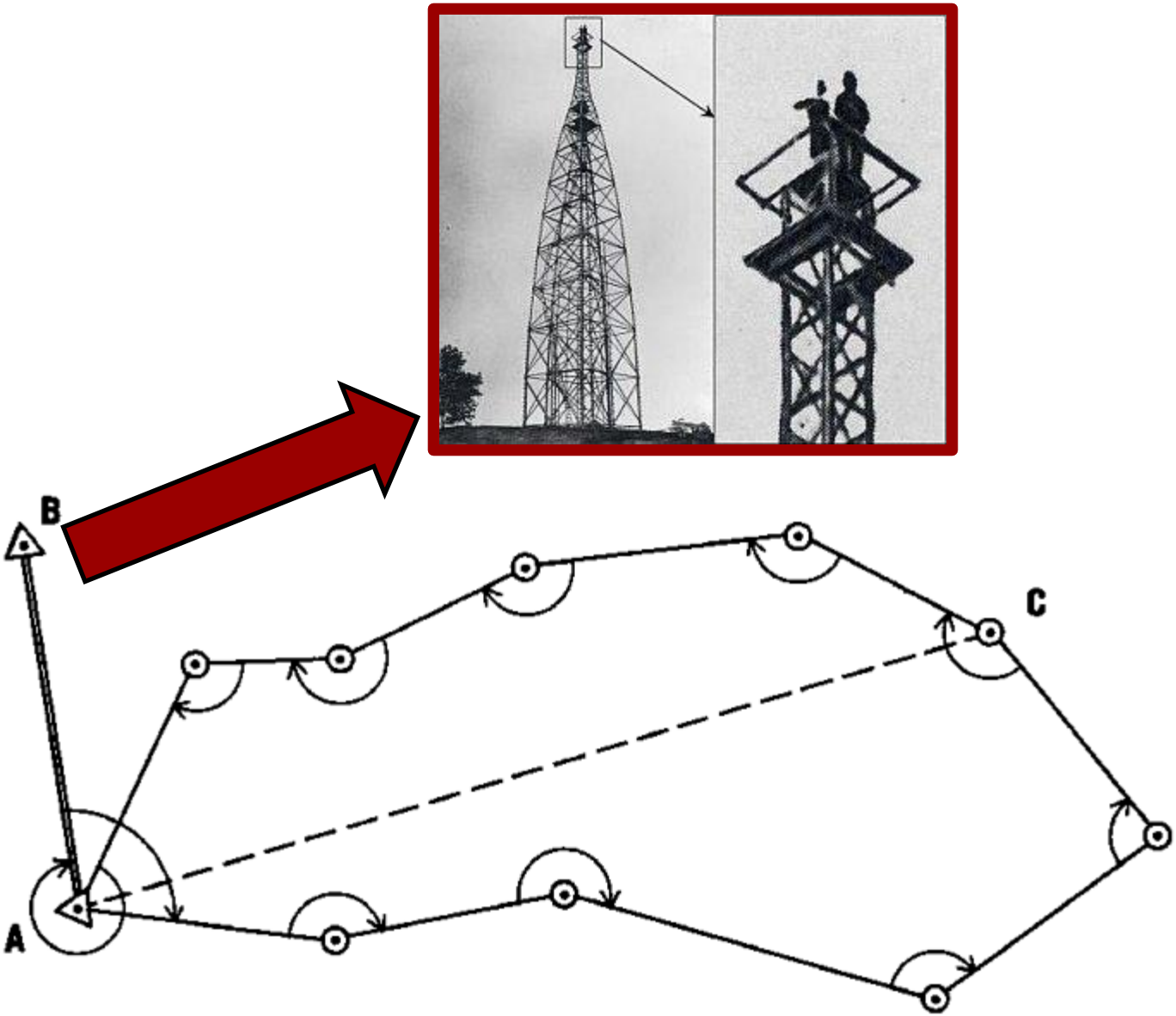
What is Geodetics?

- Measurement and representation of earth
- Survey earth at global and local scale
- Geodetic mapping

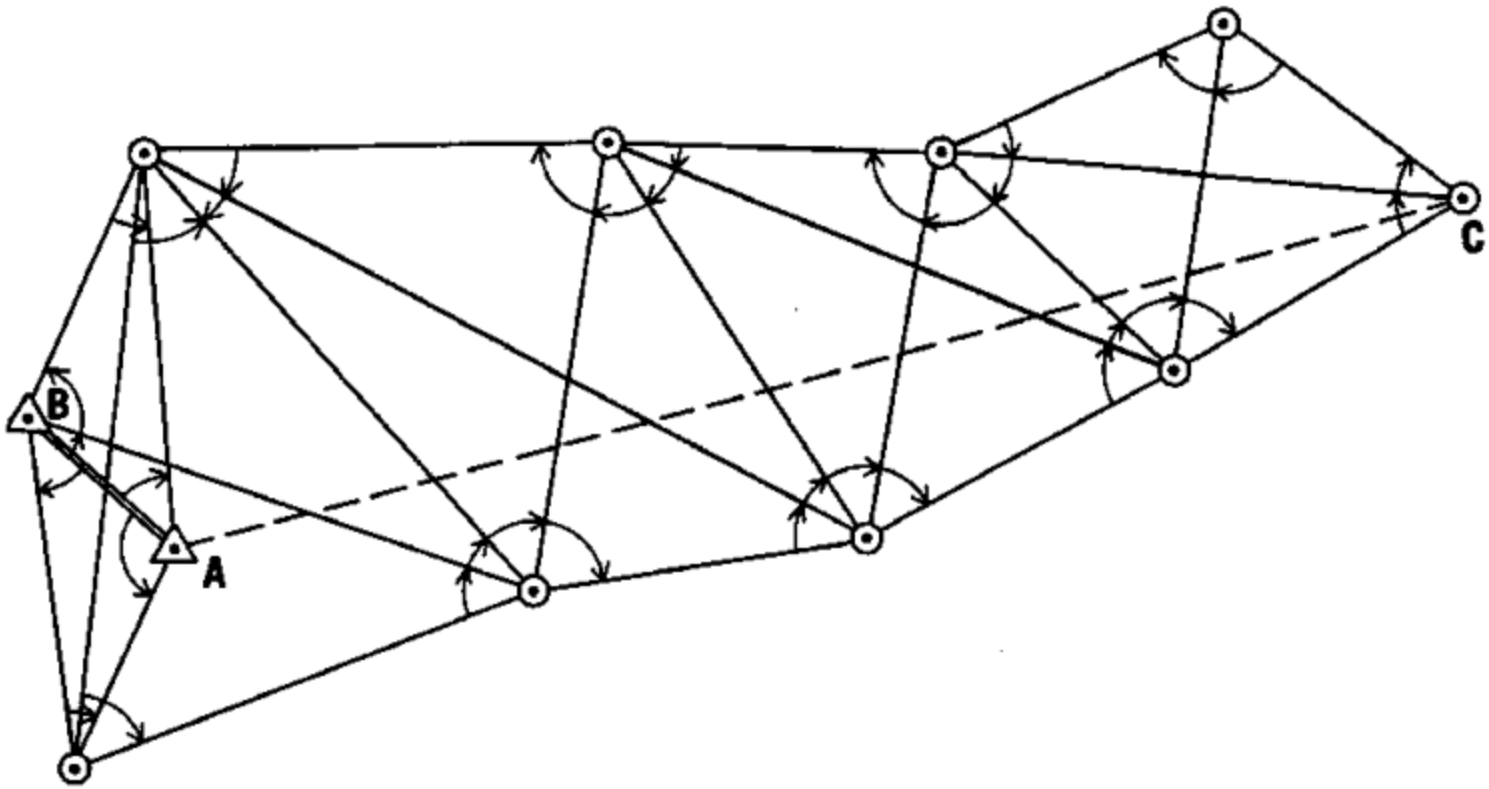
Geodetic Mapping: Triangle Net



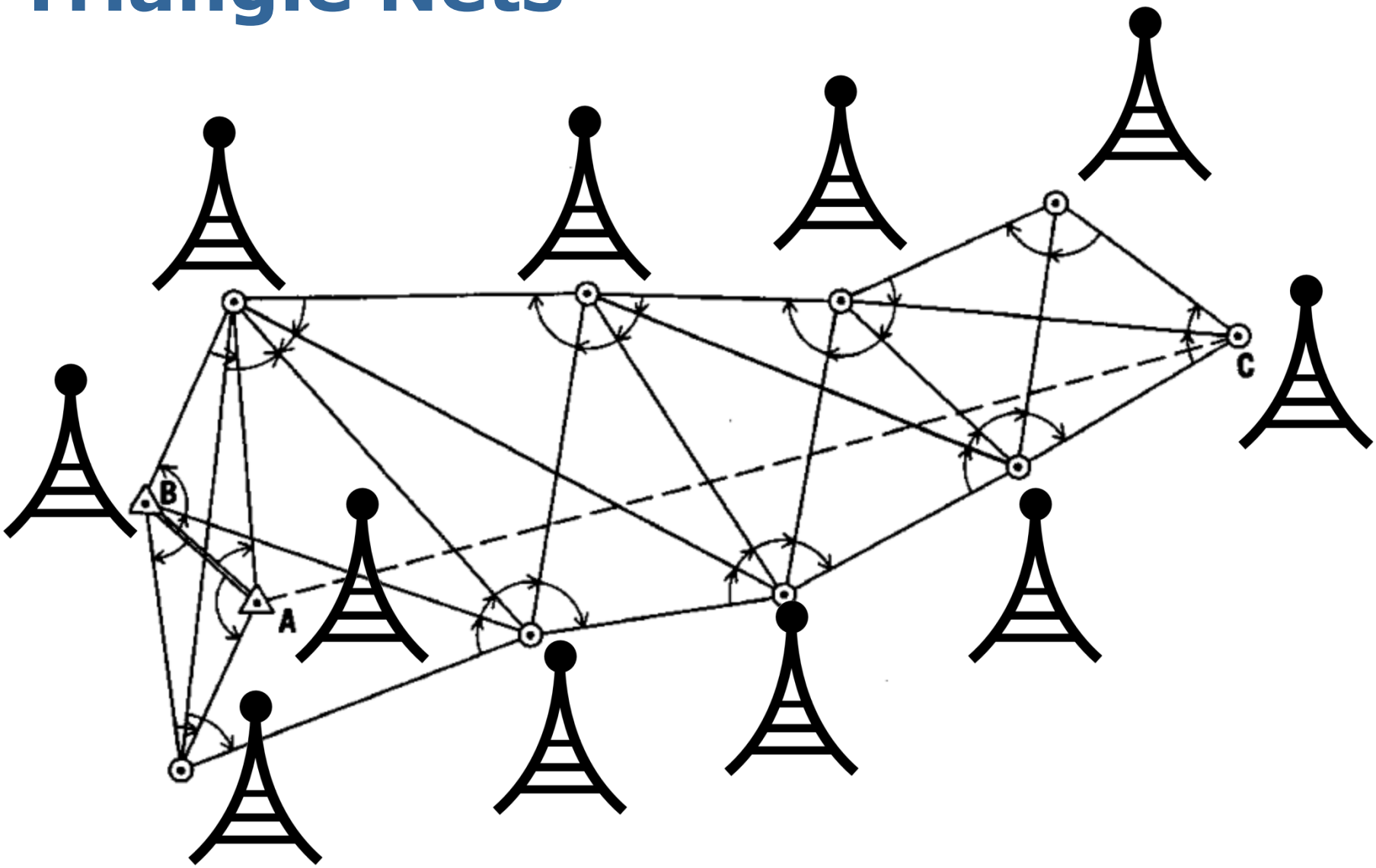
Geodetic Mapping: Triangle Net



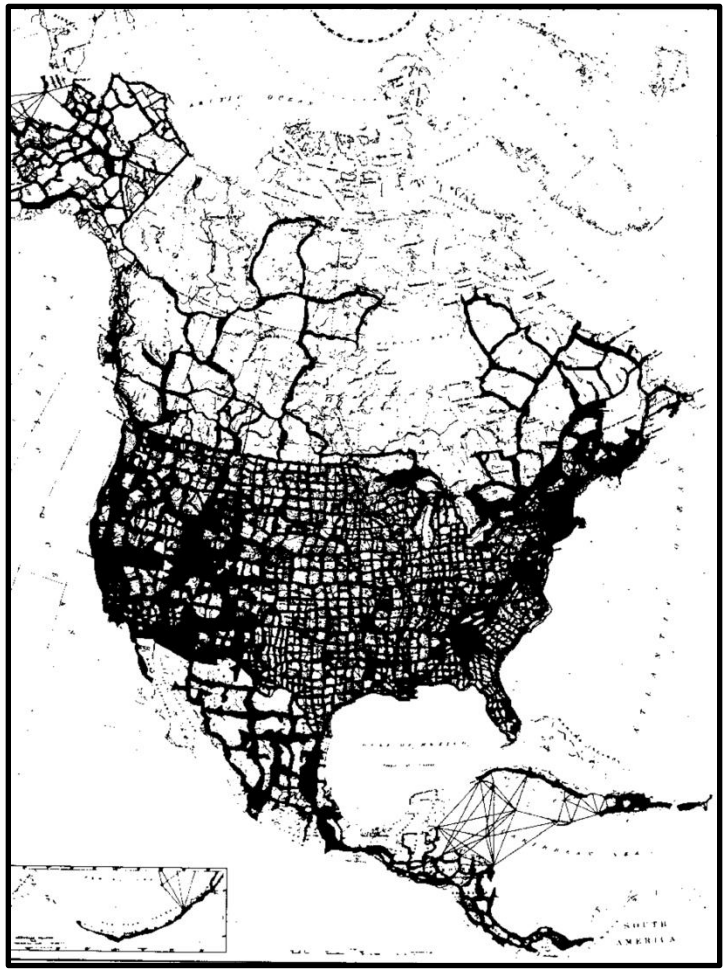
Triangle Nets



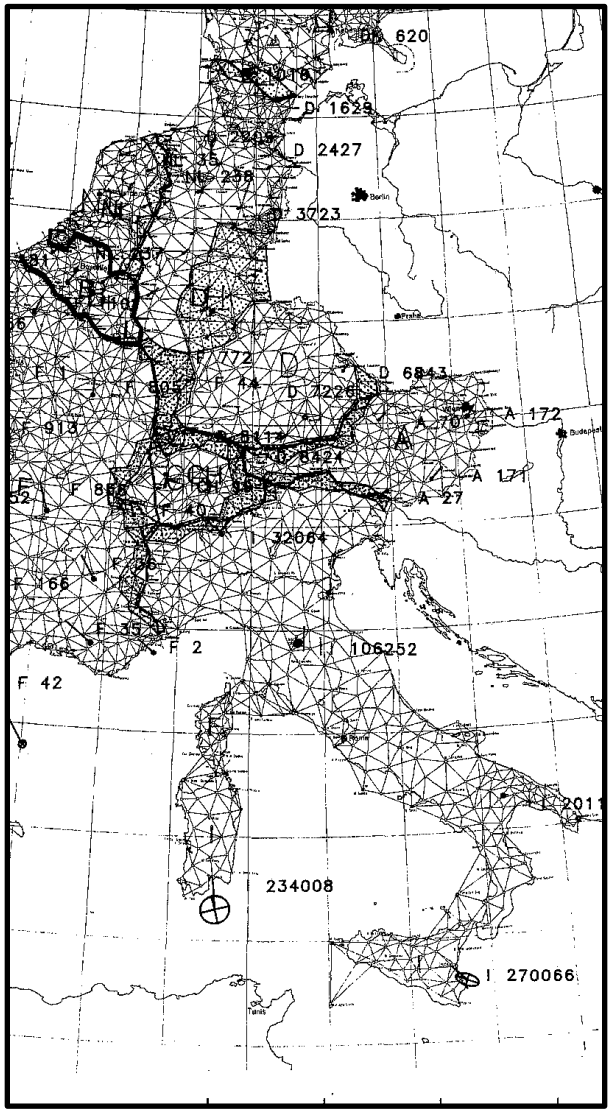
Triangle Nets



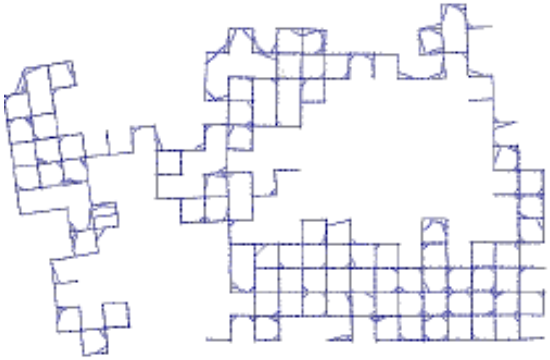
Triangle Nets

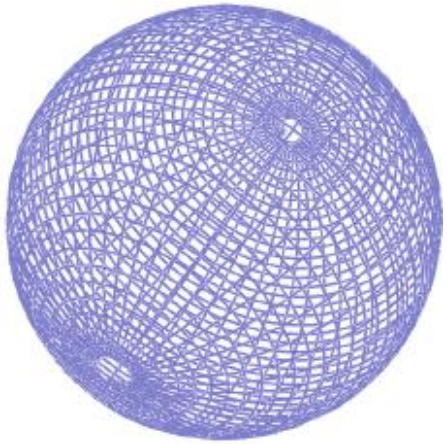


Triangle Nets



Geodetic & SLAM Graphs





Common Challenges

- Large optimization problem

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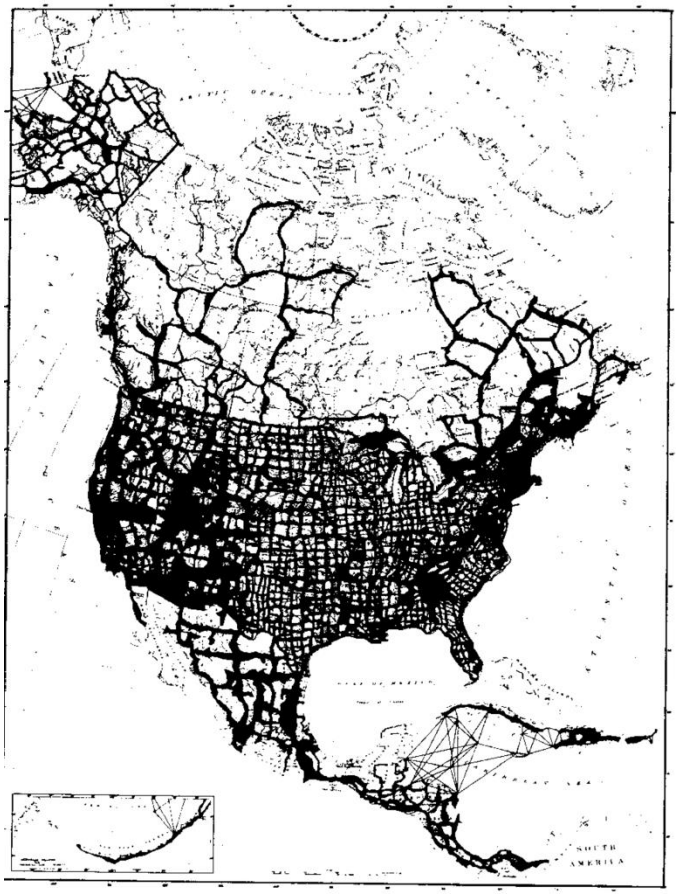
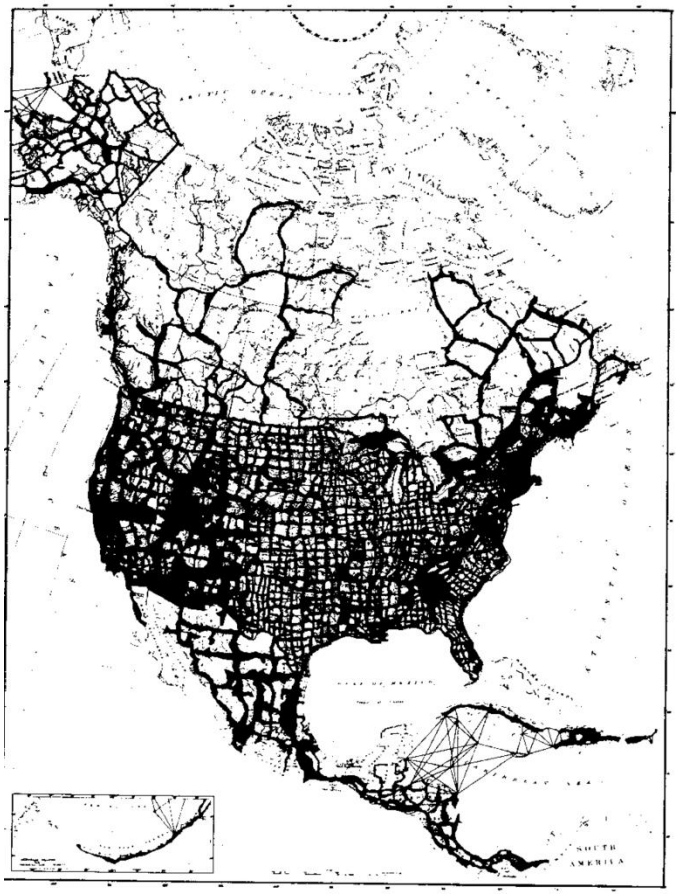


North American Datum of 1983

272,000 Stations
1,785,000 Observations

Common Challenges

- Large optimization problem



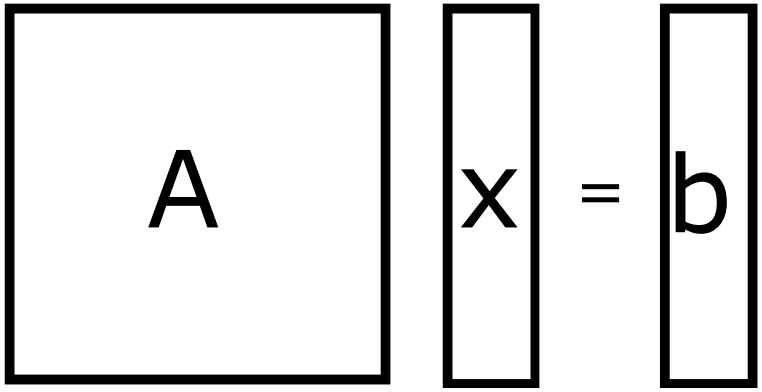
North American Datum of 1983

272,000 **Nodes**
1,785,000 **Edges**

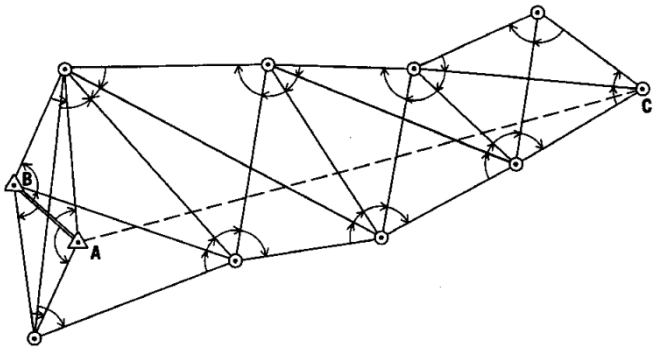
Common Challenges

- Large optimization problem
- Non-linear observations
- Front-end outliers
- Require incremental methods

Nonlinear Least Squares



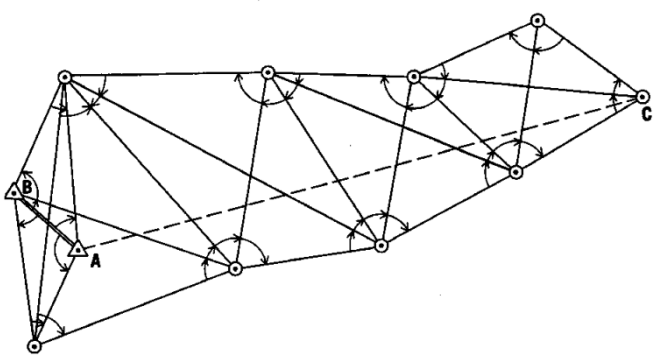
Towers



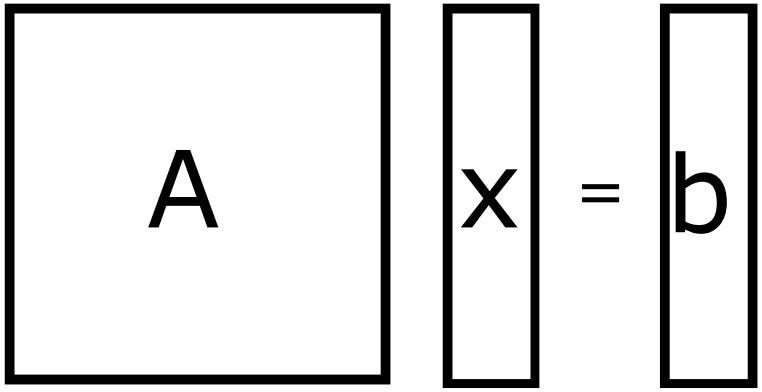
Nonlinear Least Squares

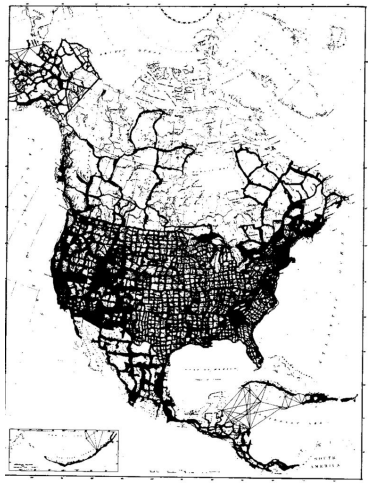
$$A \; x = b$$

Observations



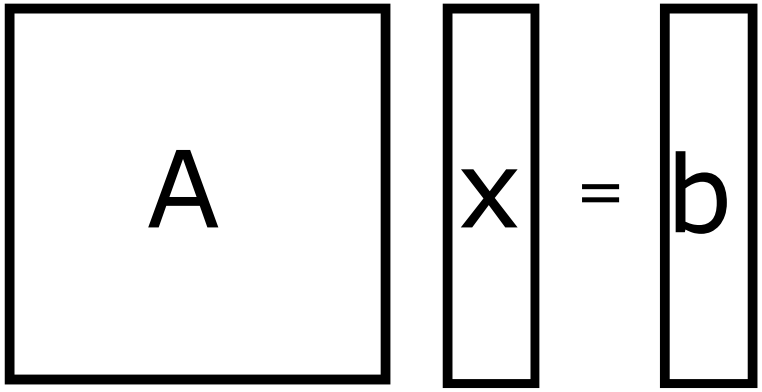
Nonlinear Least Squares

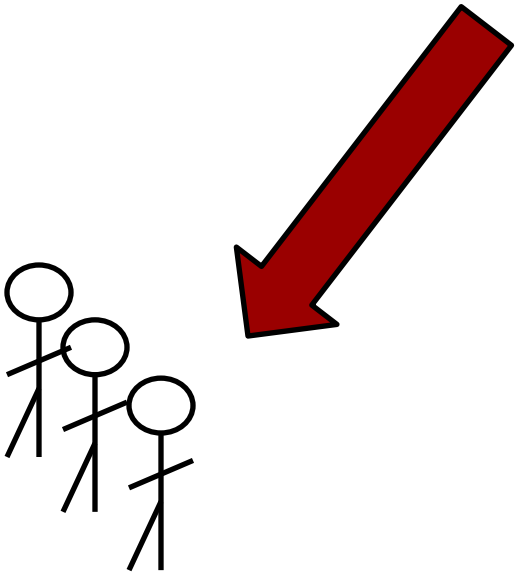


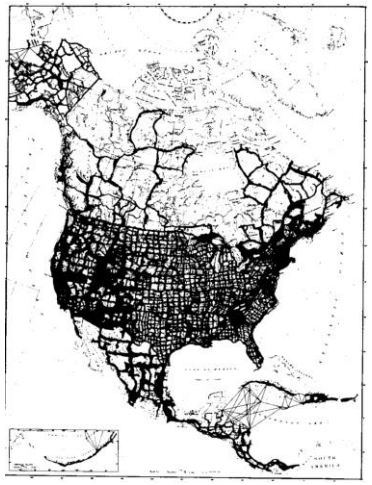


272,000 **Nodes**
1,785,000 **Edges**

Nonlinear Least Squares







272,000 **Nodes**
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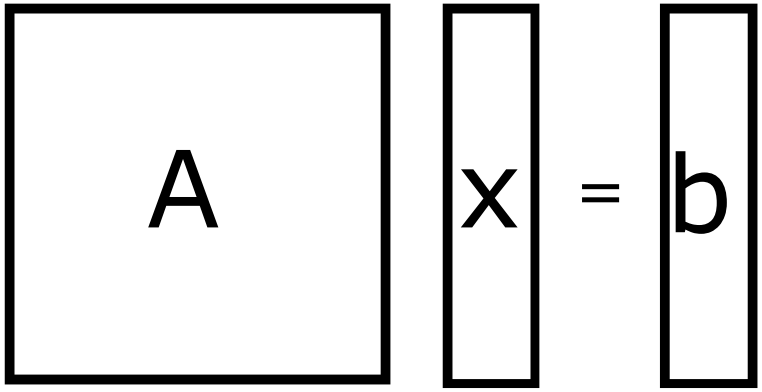
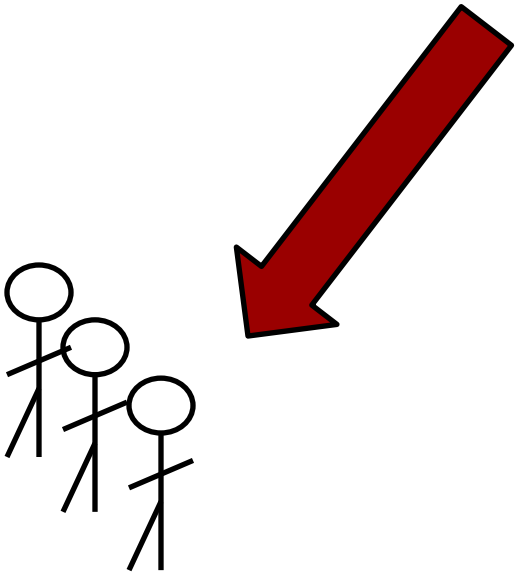
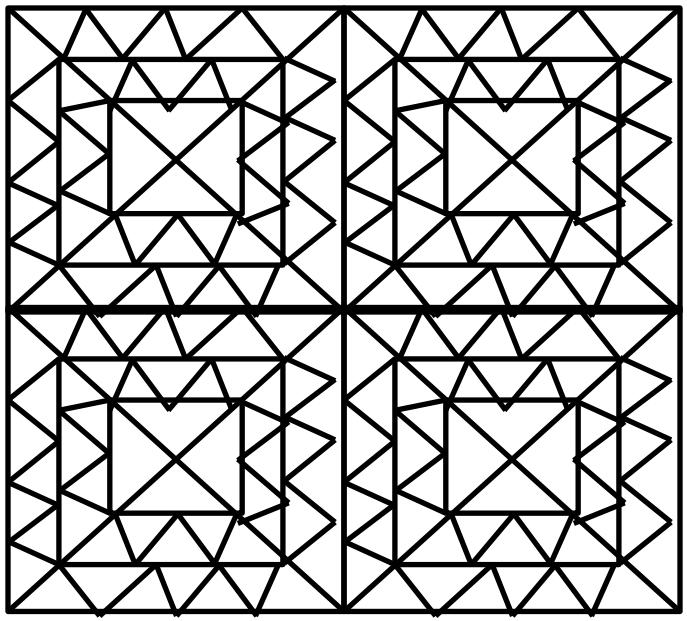
Nonlinear Least Squares

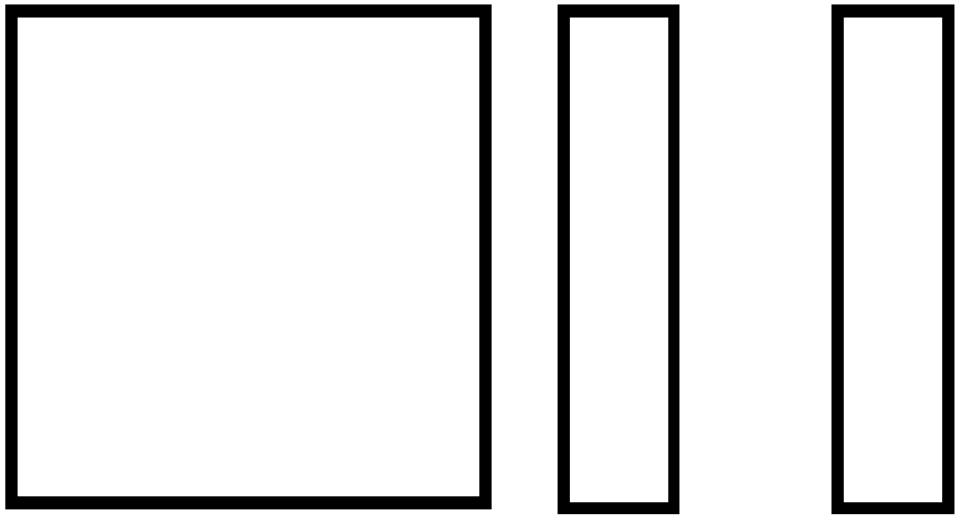




Solution:
Helmert Blocking

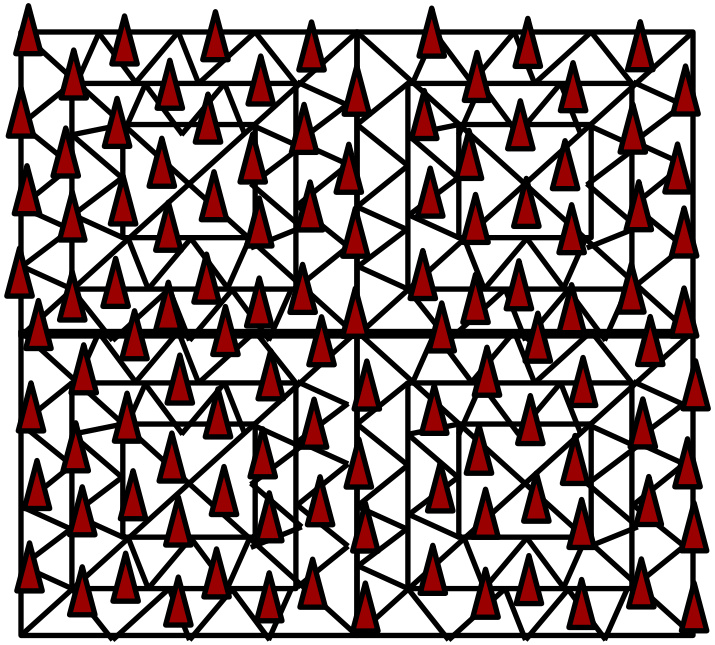
Helmert Blocking 1880

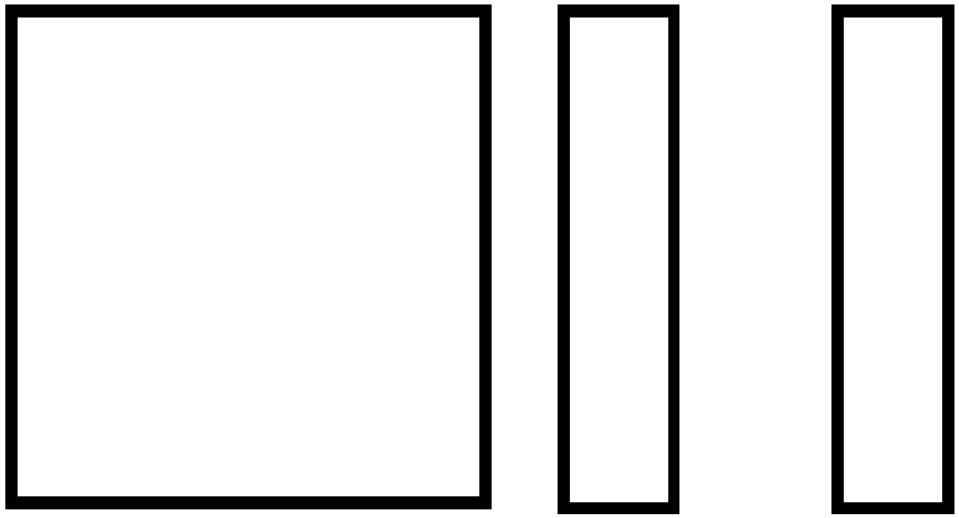




A x = b

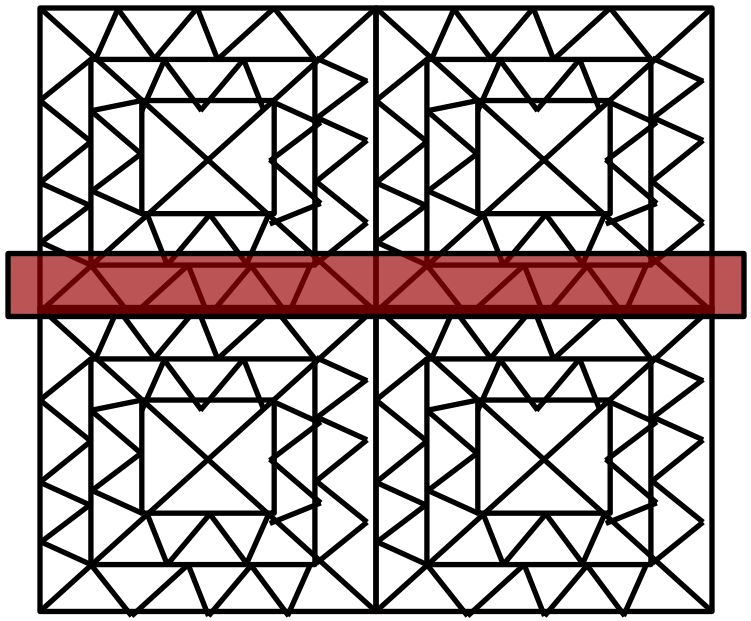
Helmert Blocking 1880

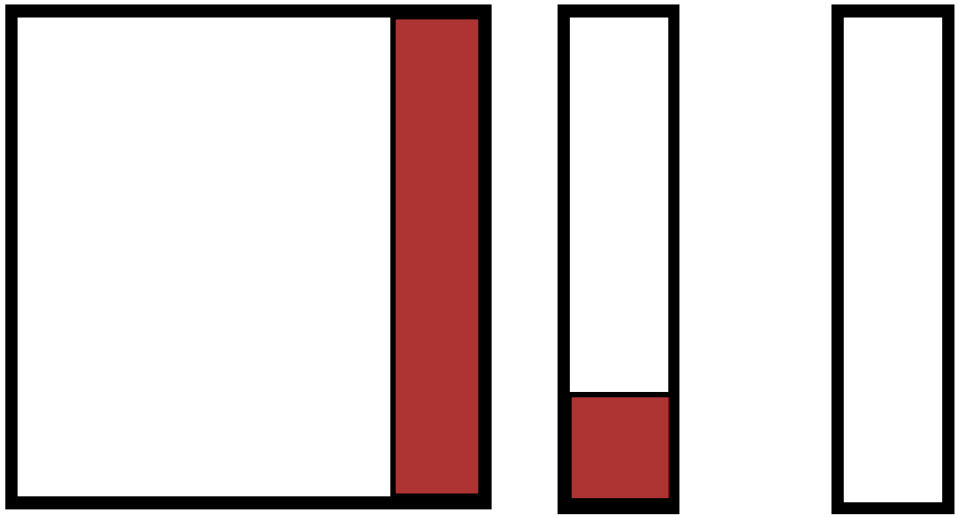




A x = b

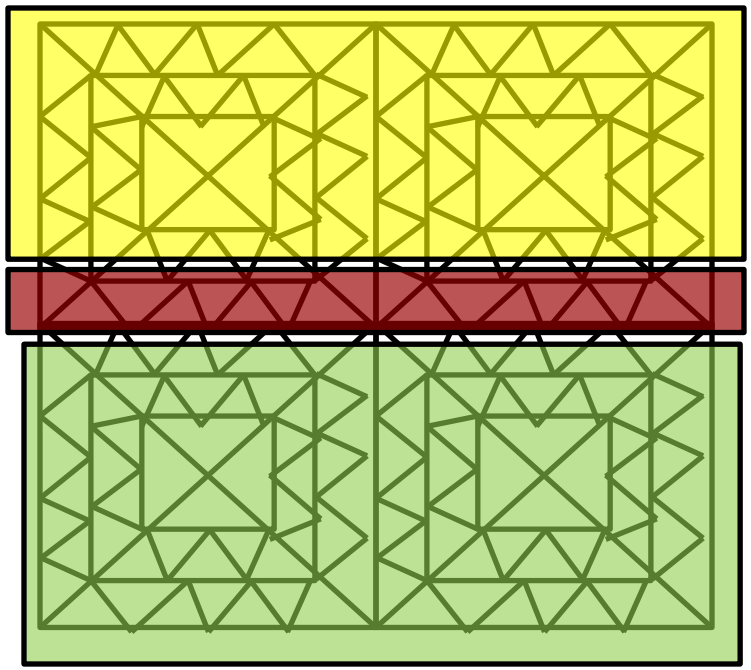
Helmert Blocking 1880

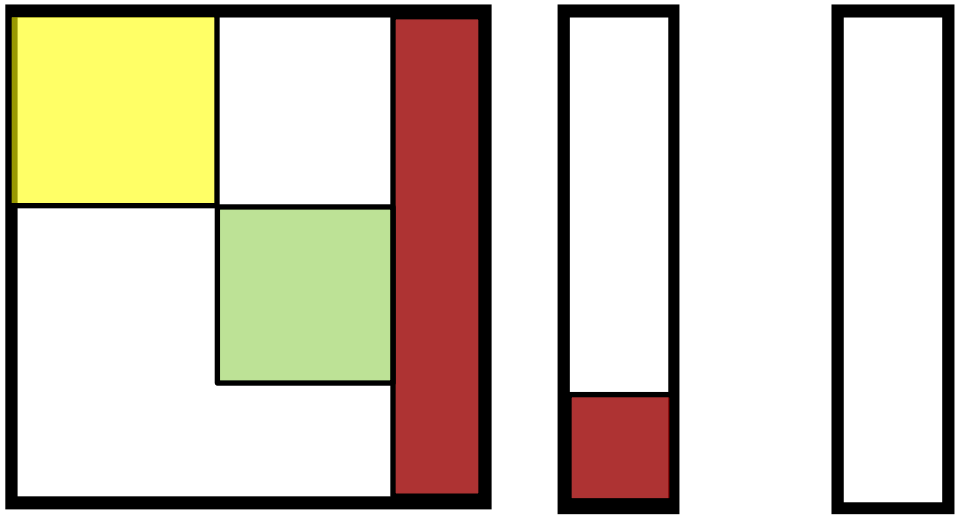




A x = b

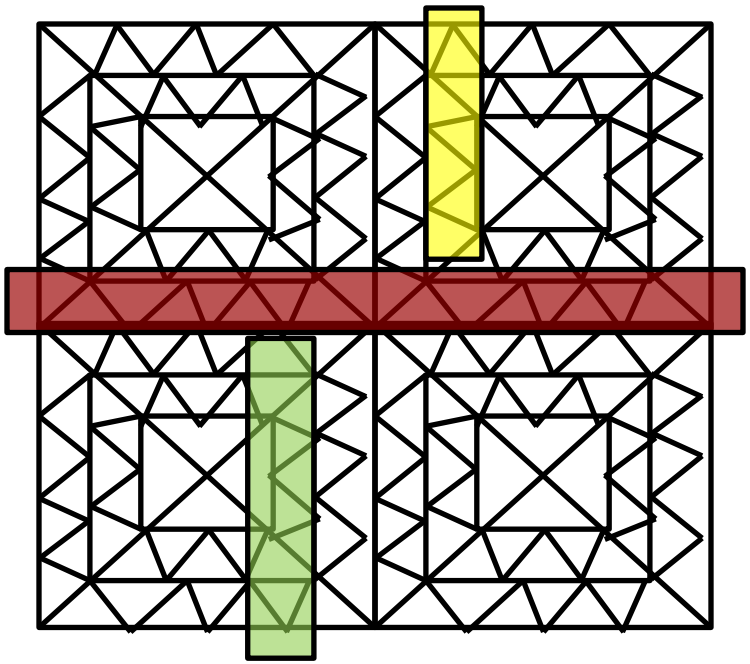
Helmert Blocking 1880

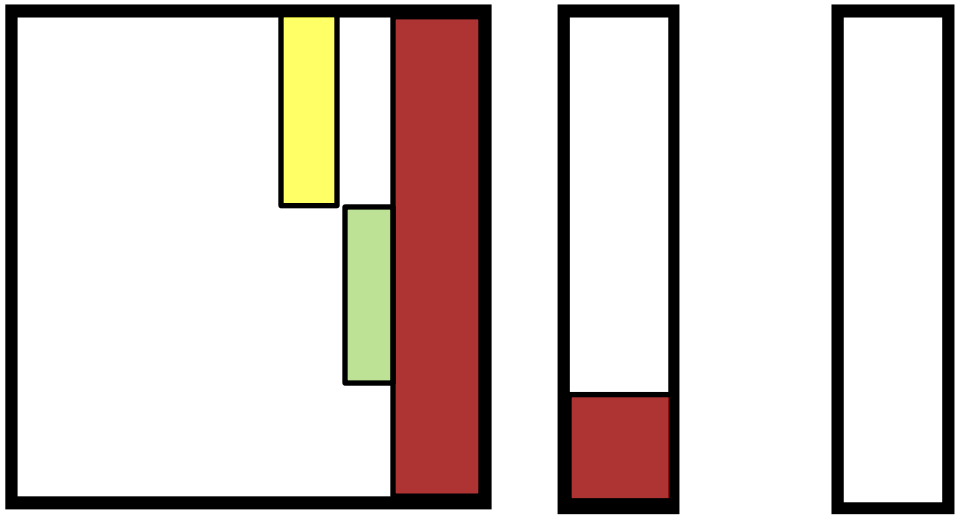




A x = b

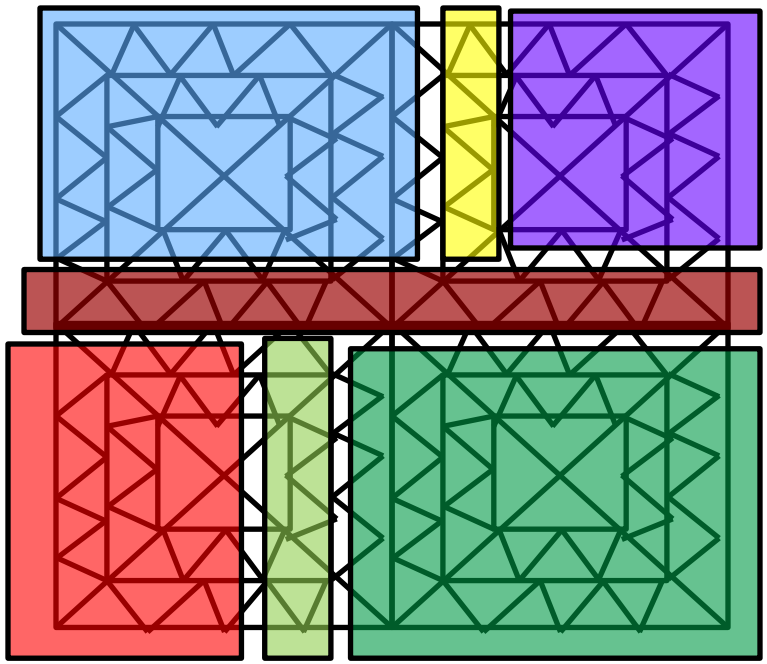
Helmert Blocking 1880

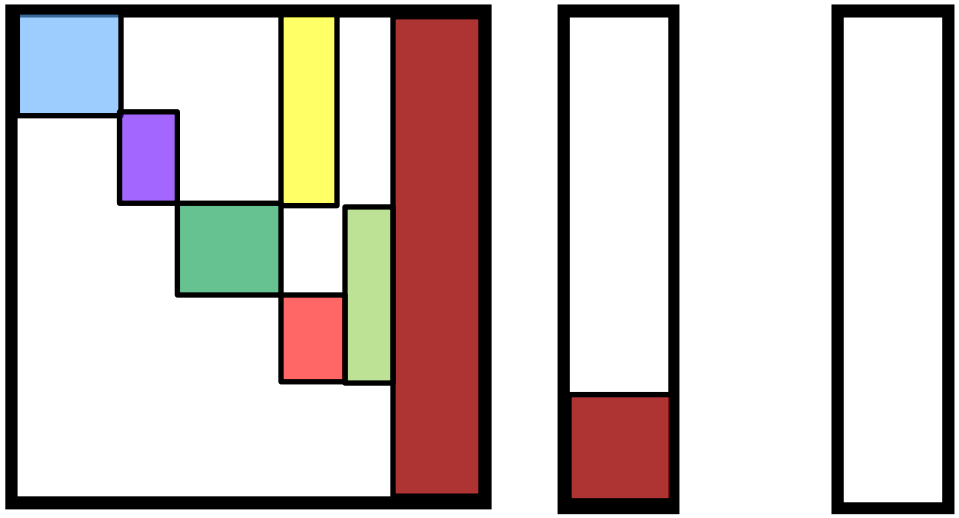




A x = b

Helmert Blocking 1880

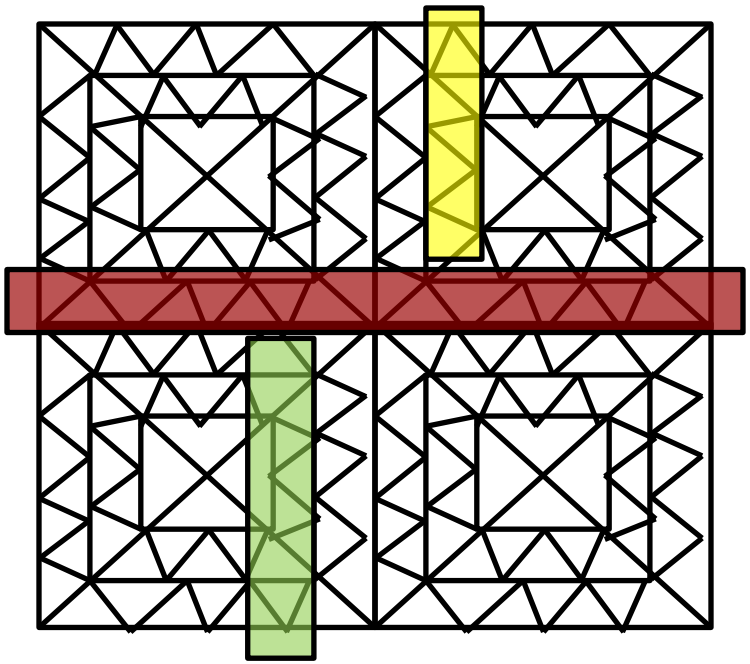


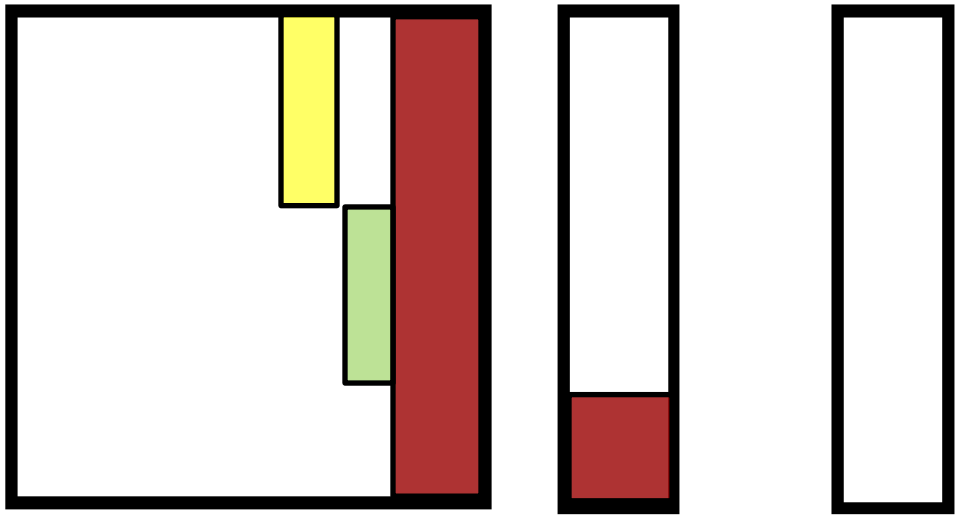


A x = b

Exact method

Helmert Blocking 1880

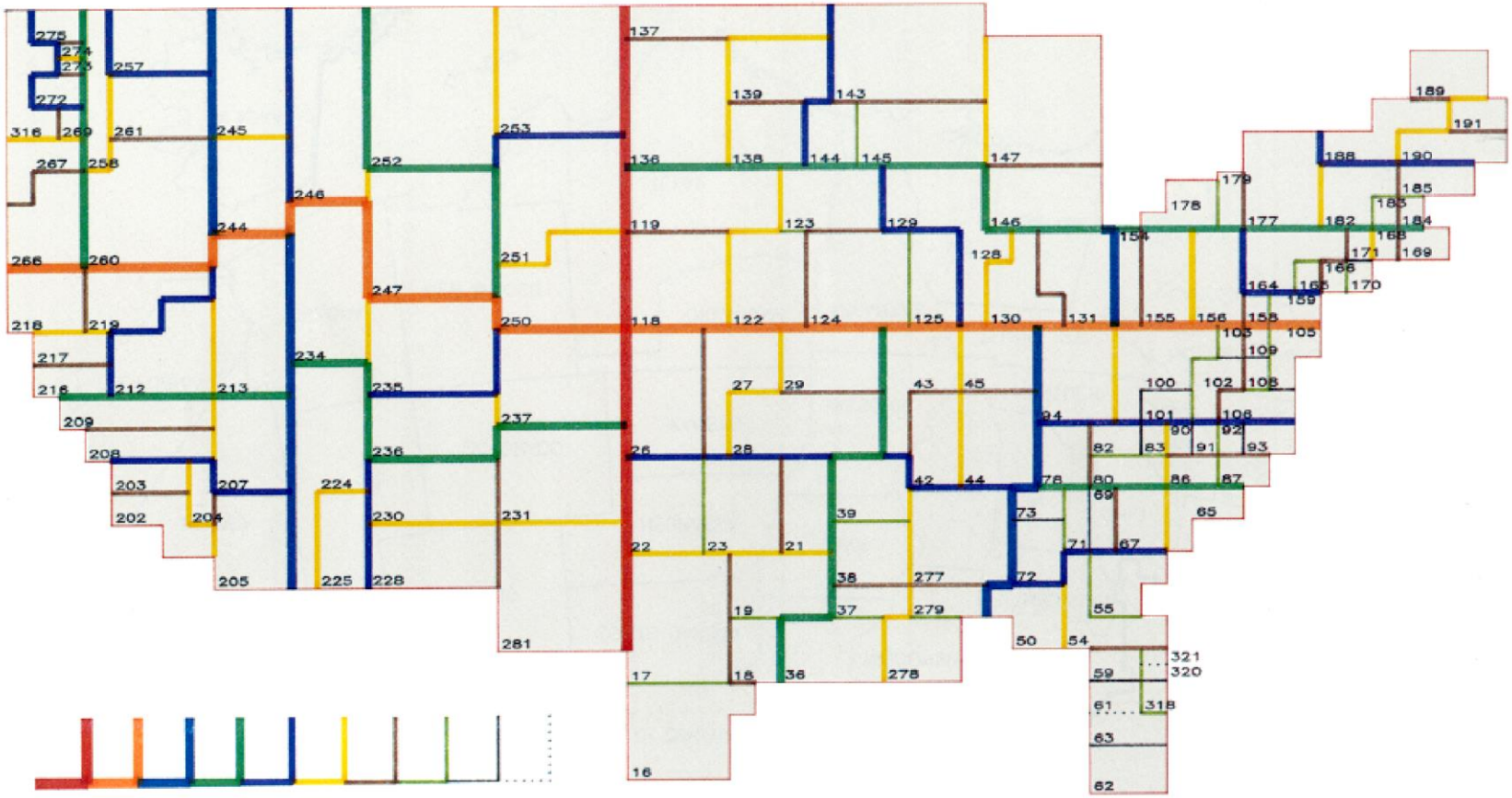




A x = b

The order of cuts is spatial along lat, long

Helmert’s Blocks

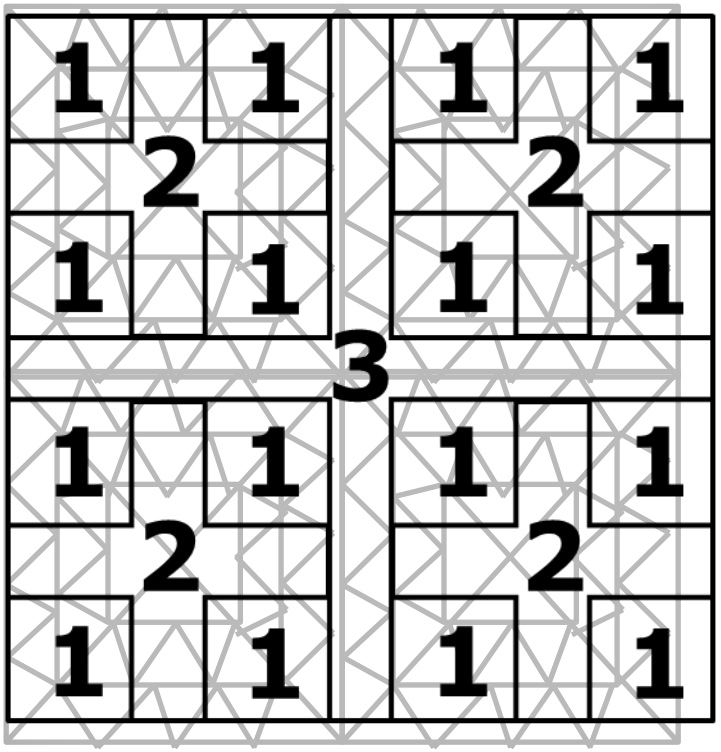


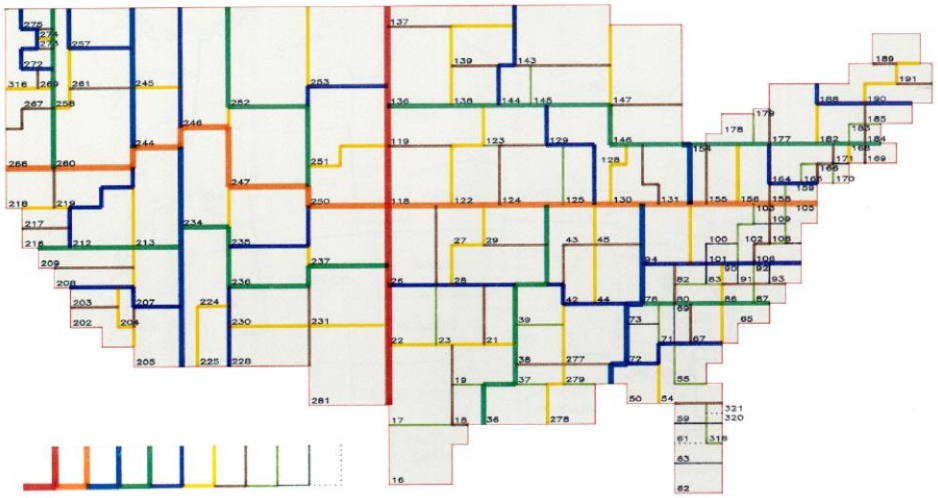
Helmert's Ordering for Parallel Optimization

- Avila & Tomlin 1979: Parallel least square optimization with ILLIAC IV.

- Golub & Plemmons 1980: Large-scale geodetic least-squares adjustment using QR decomposition.

Related to Nested Dissection

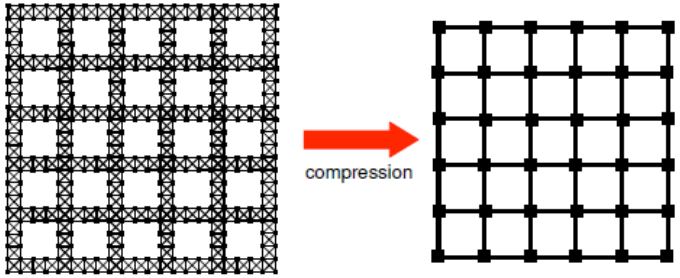




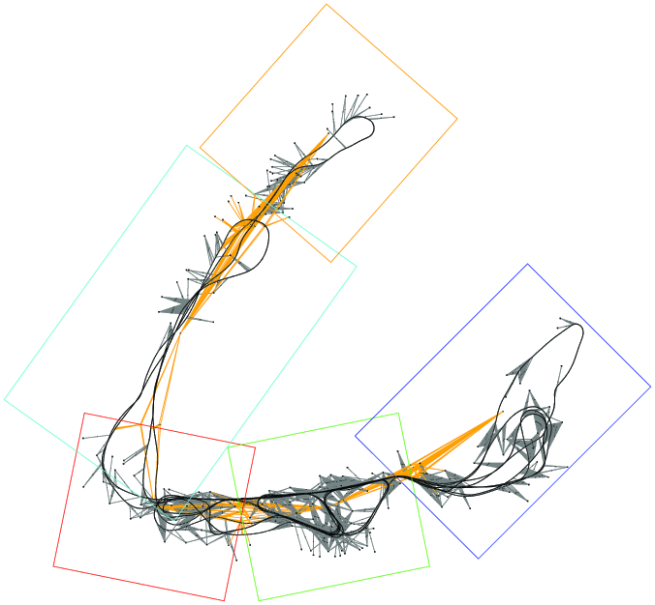
Avila & Tomlin, 1979

Nested Dissection in SLAM

- NESDIS SLAM, RSS' 06

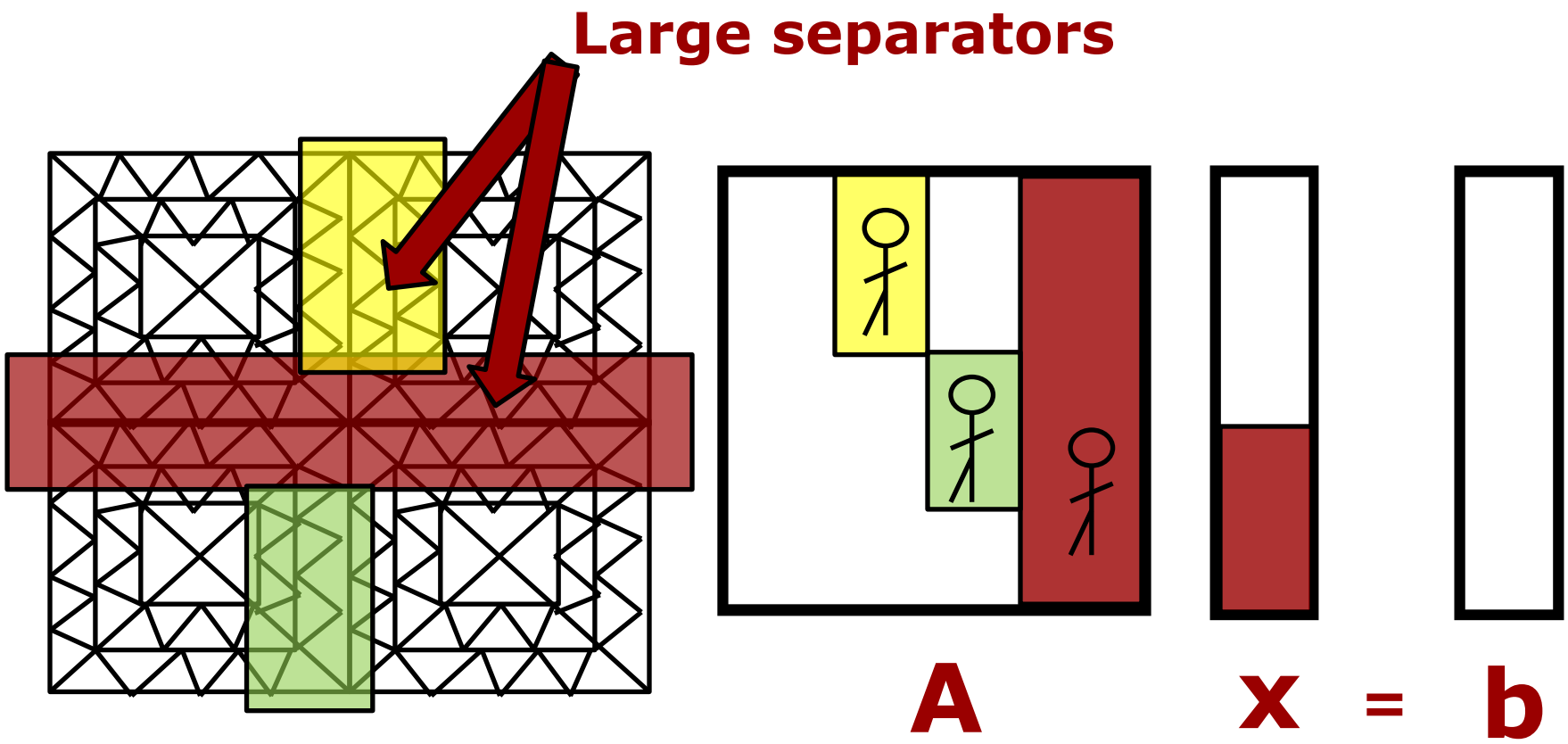


- Tectonic SAM
  ICRA' 07, IROS' 10



Images courtesy of respective authors

Issues with Helmert Blocking



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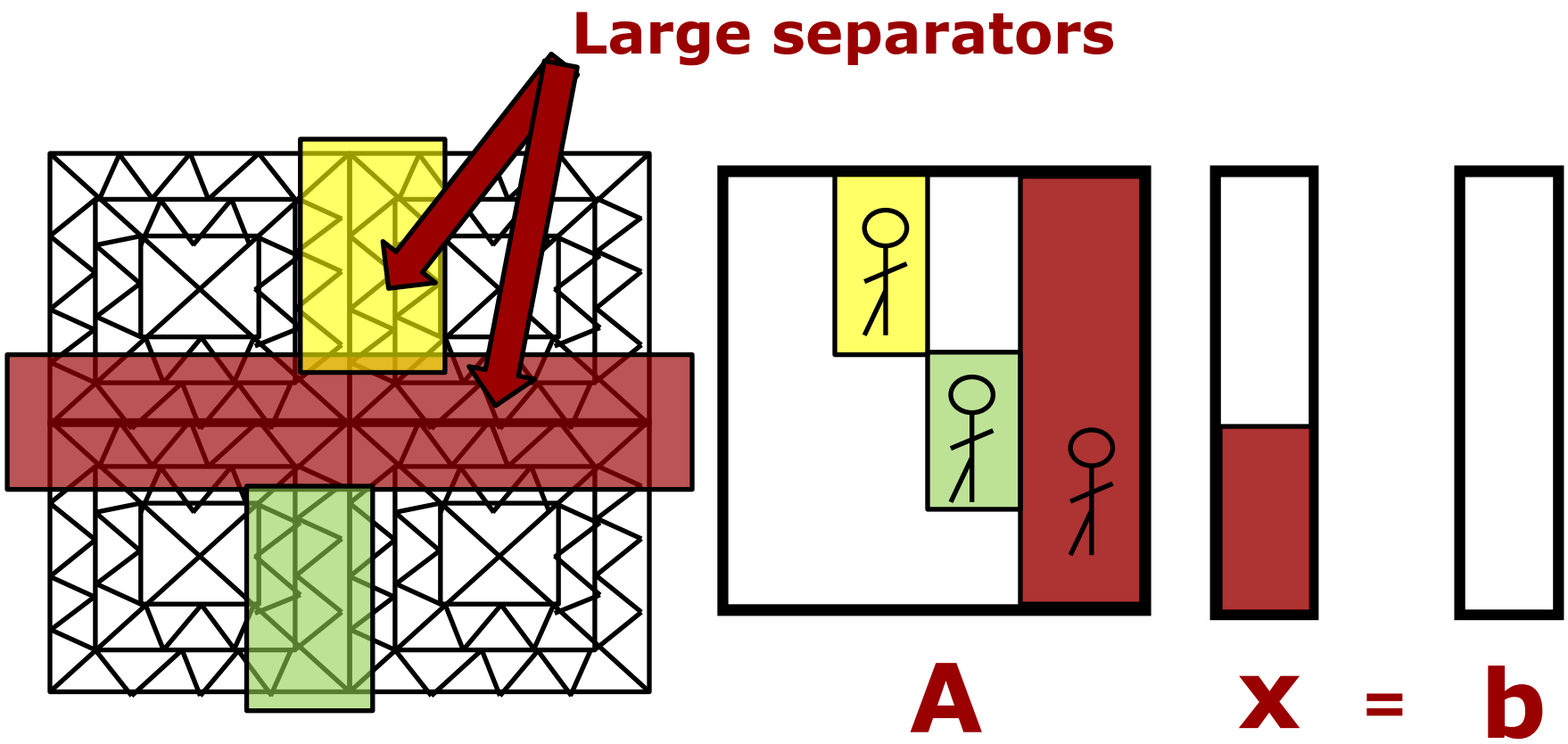
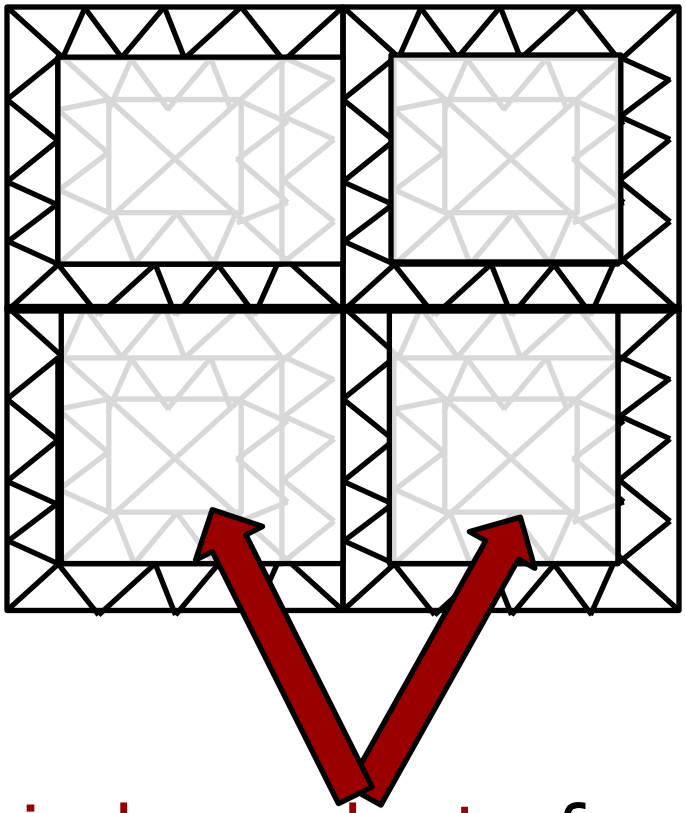


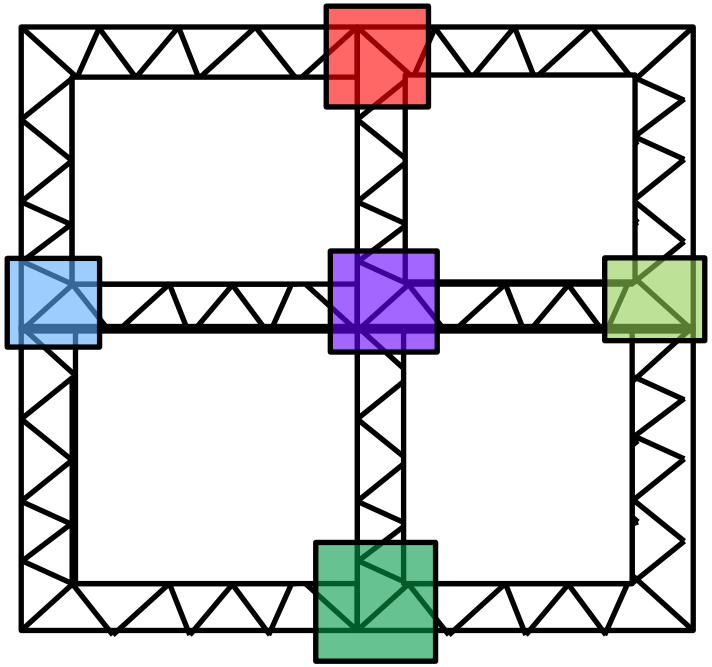
Solution: Polygon filling

Polygon Filling



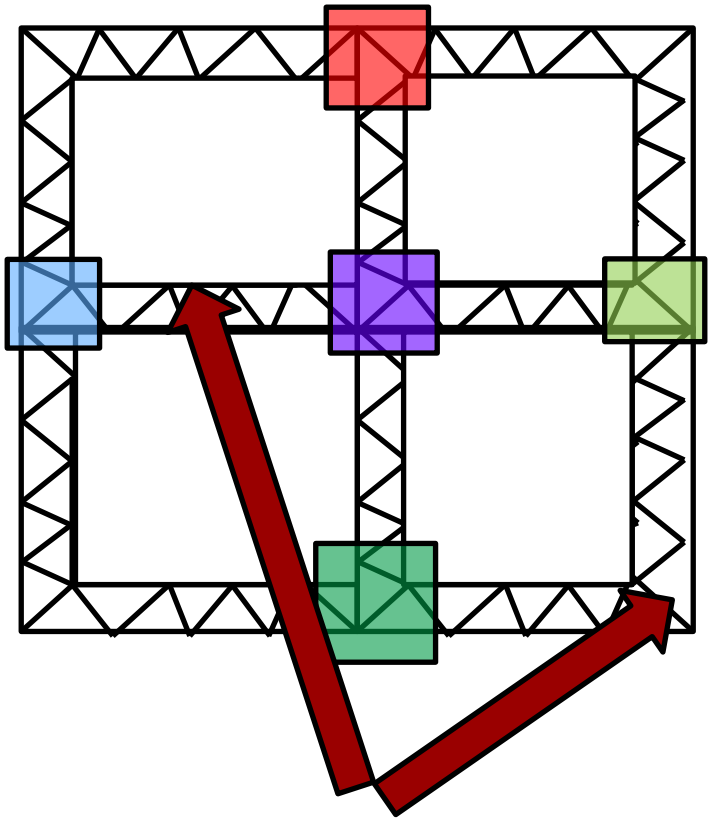
Insides are independent of each other

Polygon Filling



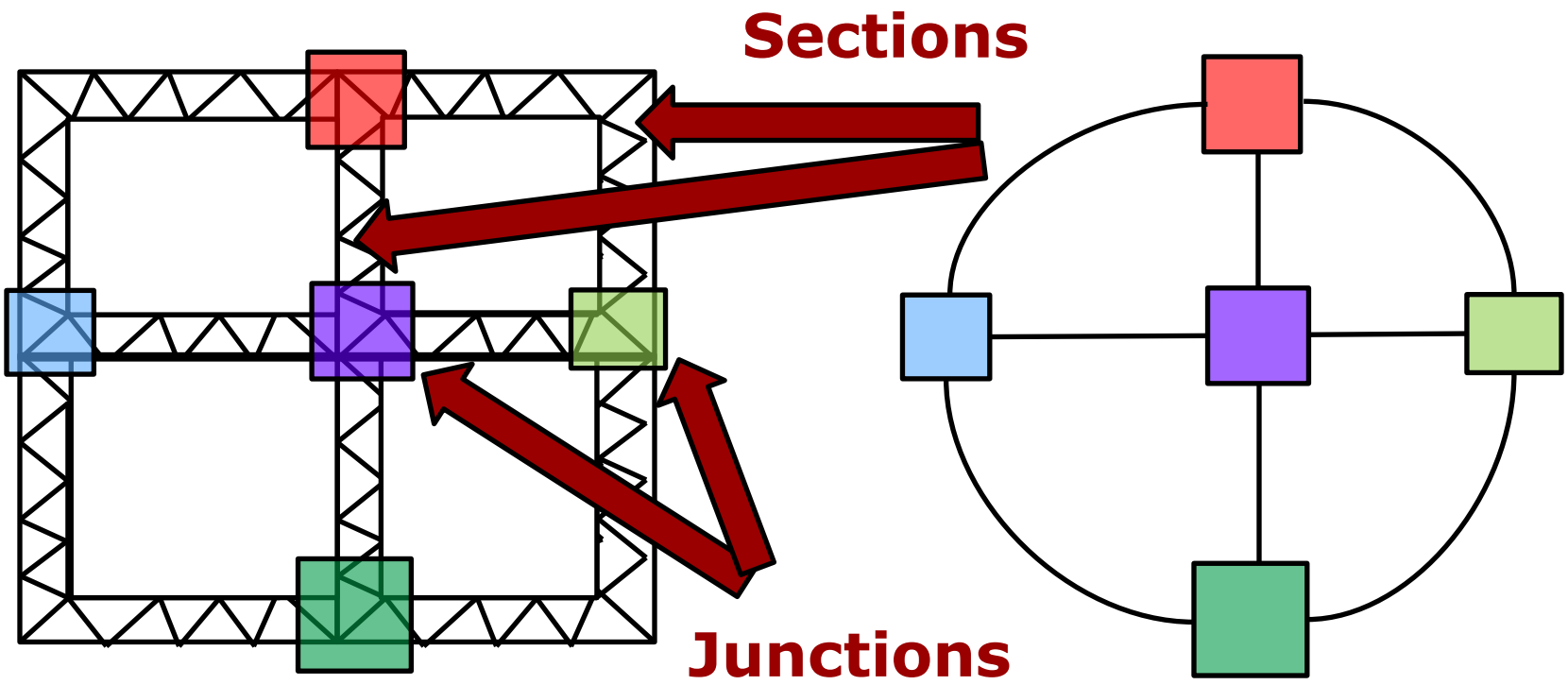
Apply Helmert’s method

Polygon Filling

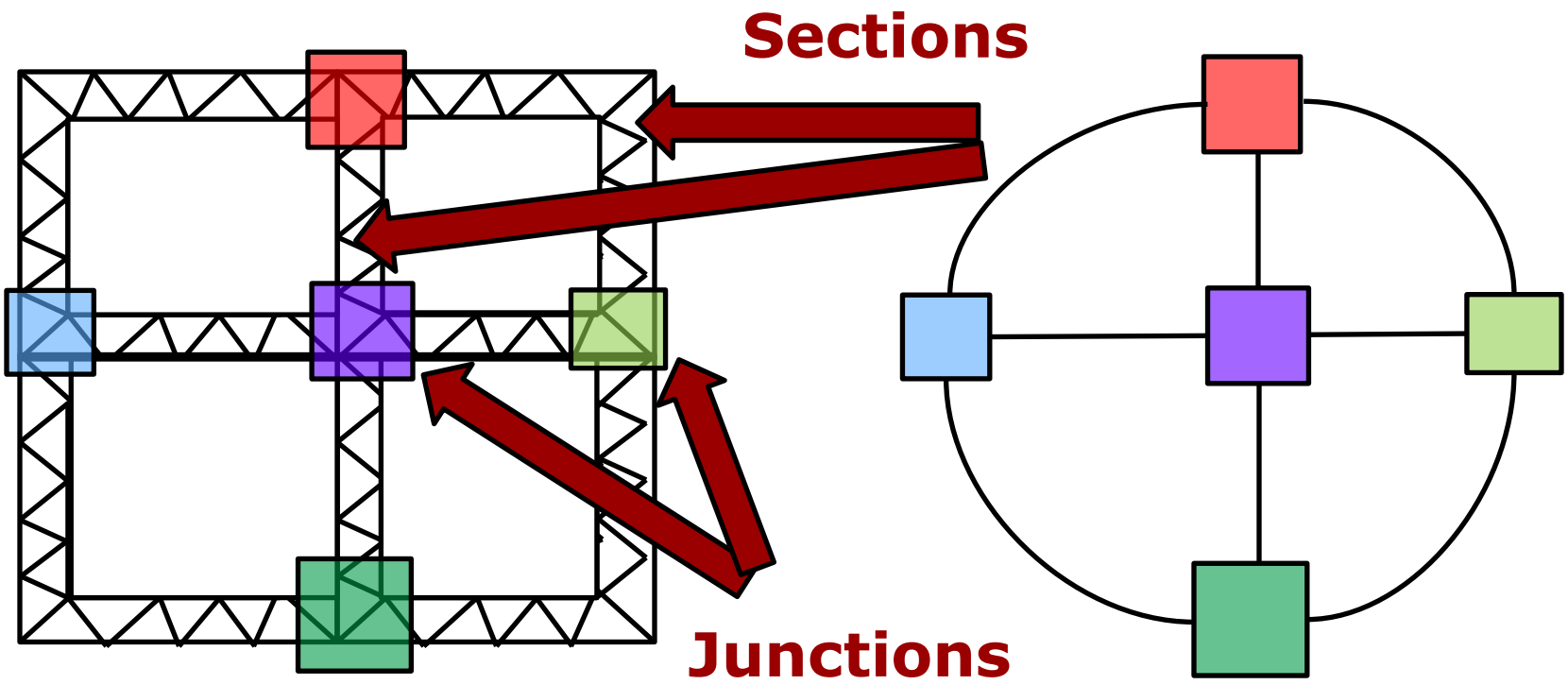


Optimization may still be too complicated!

Bowie Method of 1927

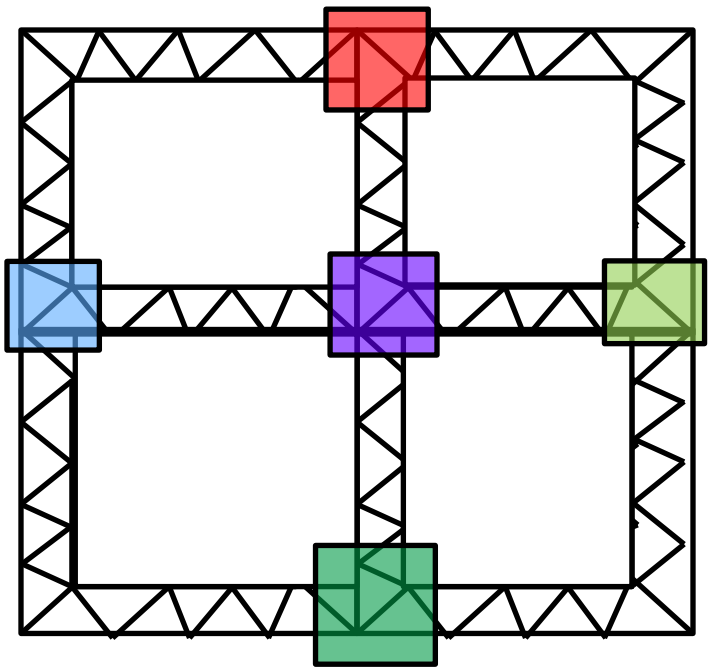


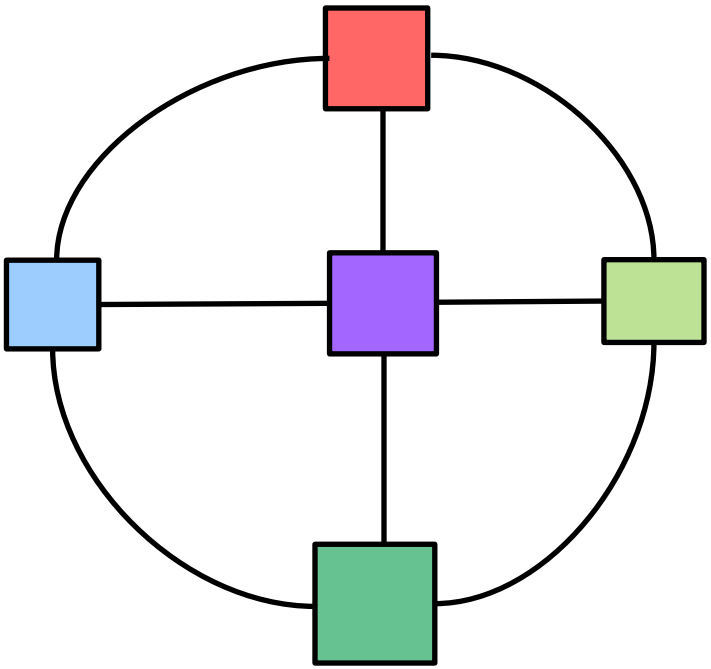
Bowie Method of 1927



- 1 virtual measurements for each section
- Solve junctions ➡ propagate to sections

Bowie Method of 1927





X Nodes

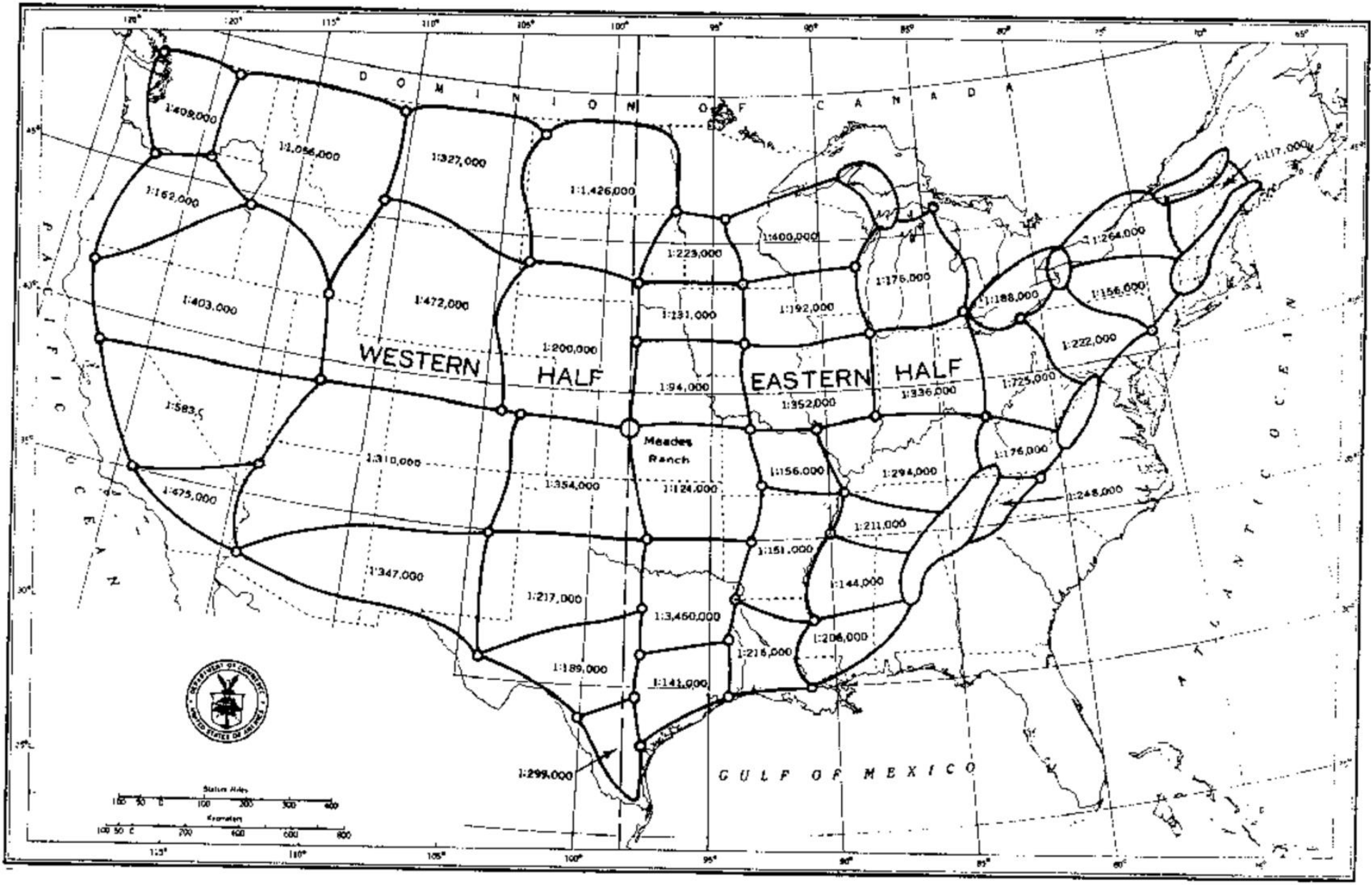
Y edges

5 Nodes

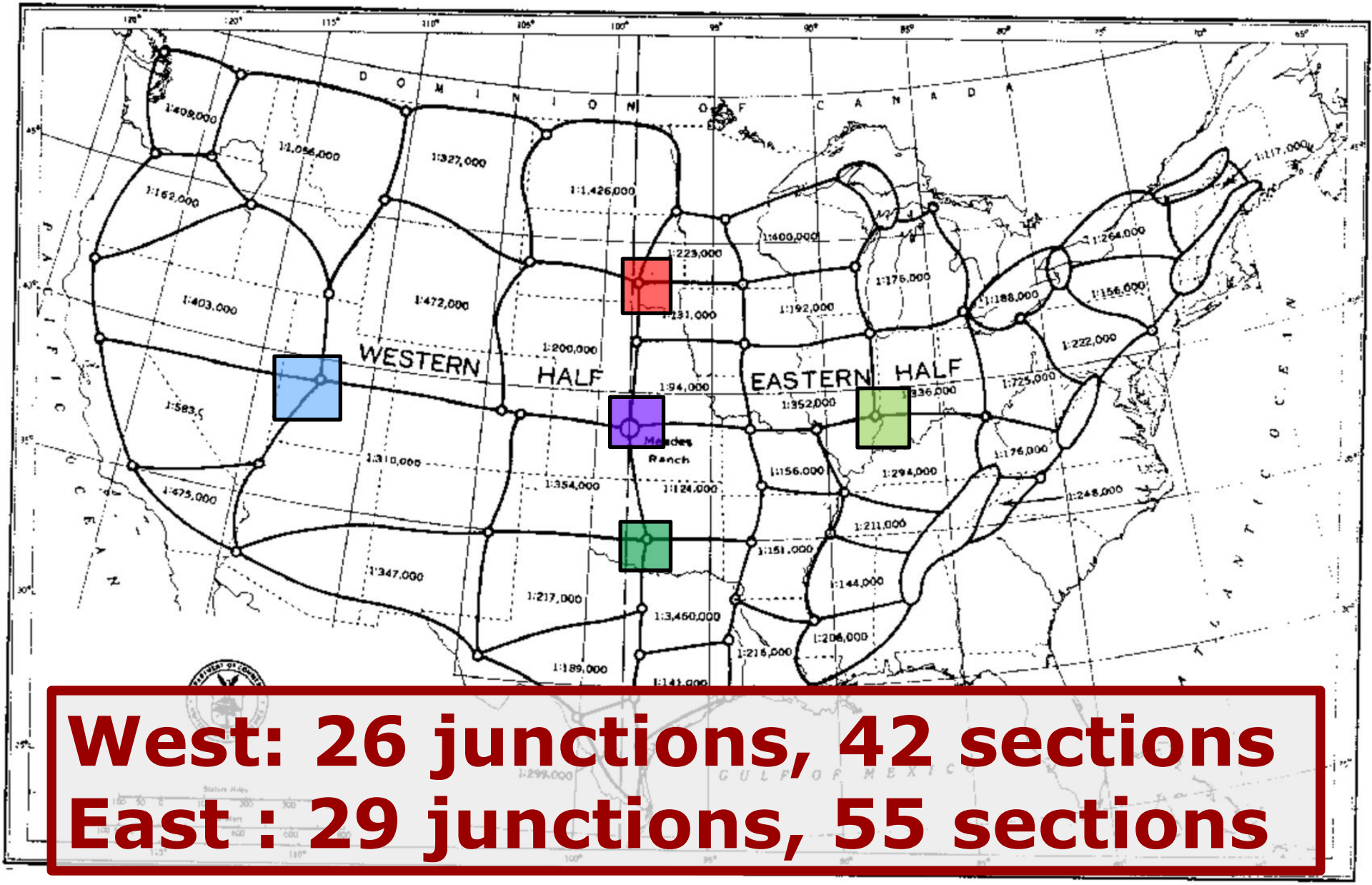
7 edges

Shares idea with Hierarchical SLAM / HOGMan

Bowie Method for NAD 1927

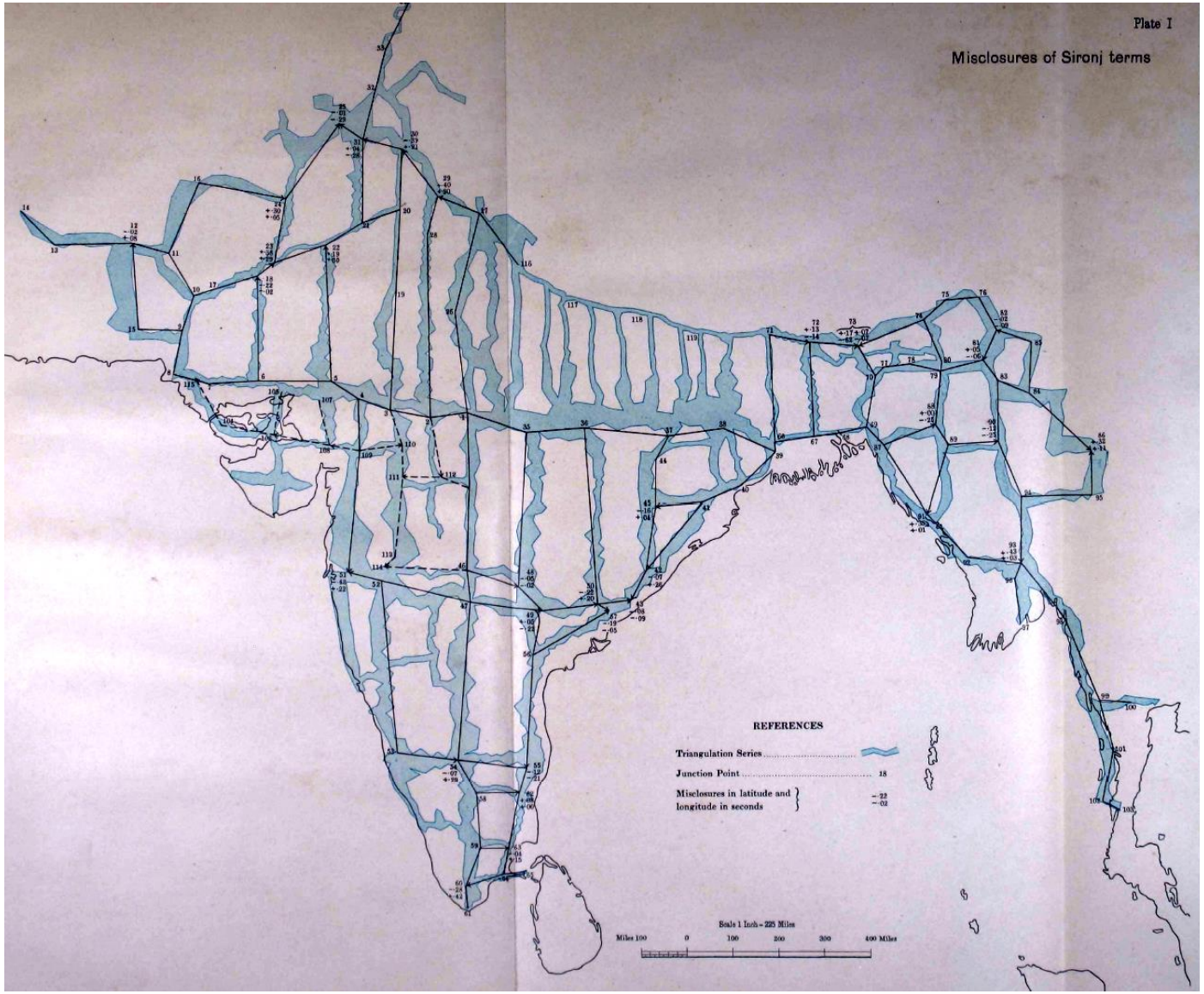


Bowie Method for NAD 1927

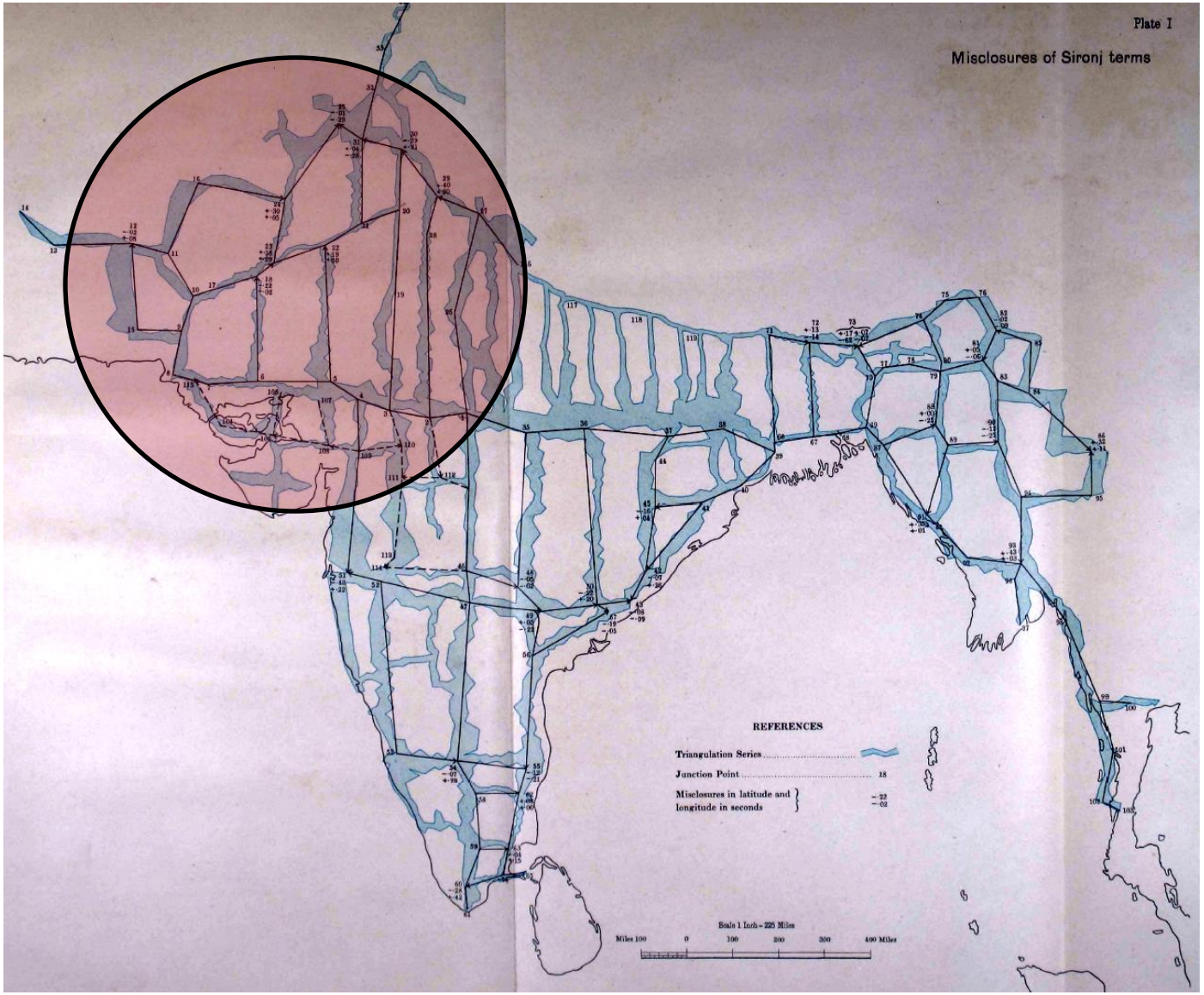


West: 26 junctions, 42 sections
East : 29 junctions, 55 sections

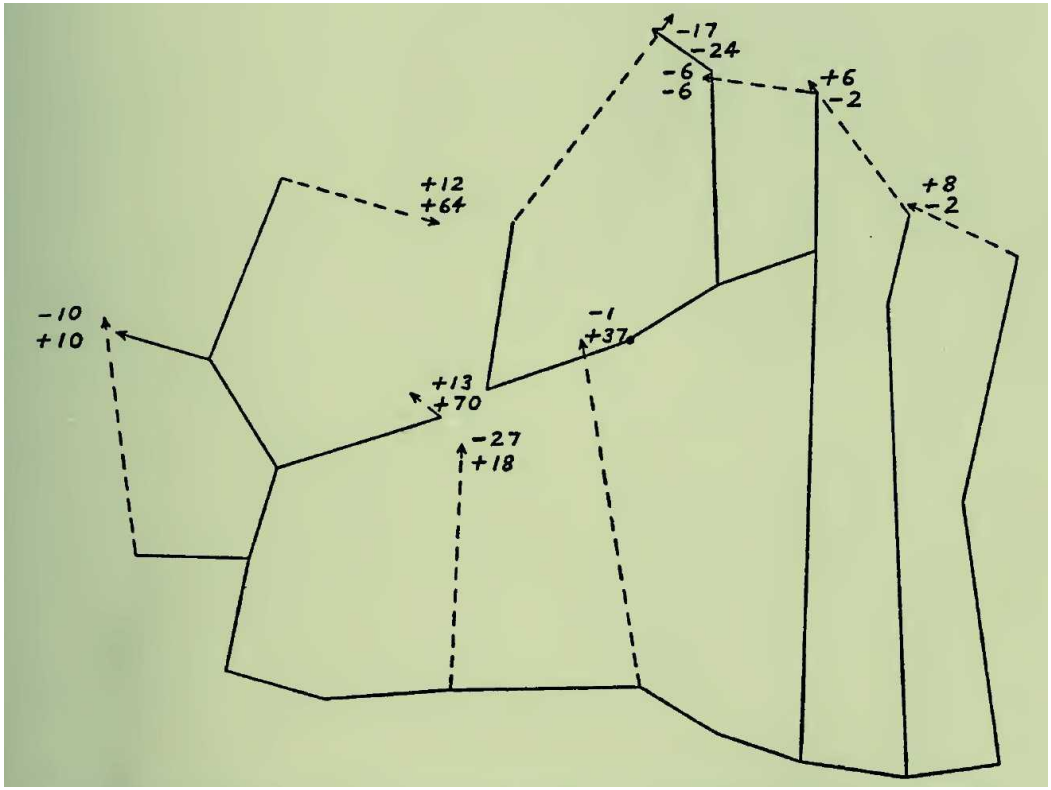
Indian Method of 1938



Indian Method of 1938

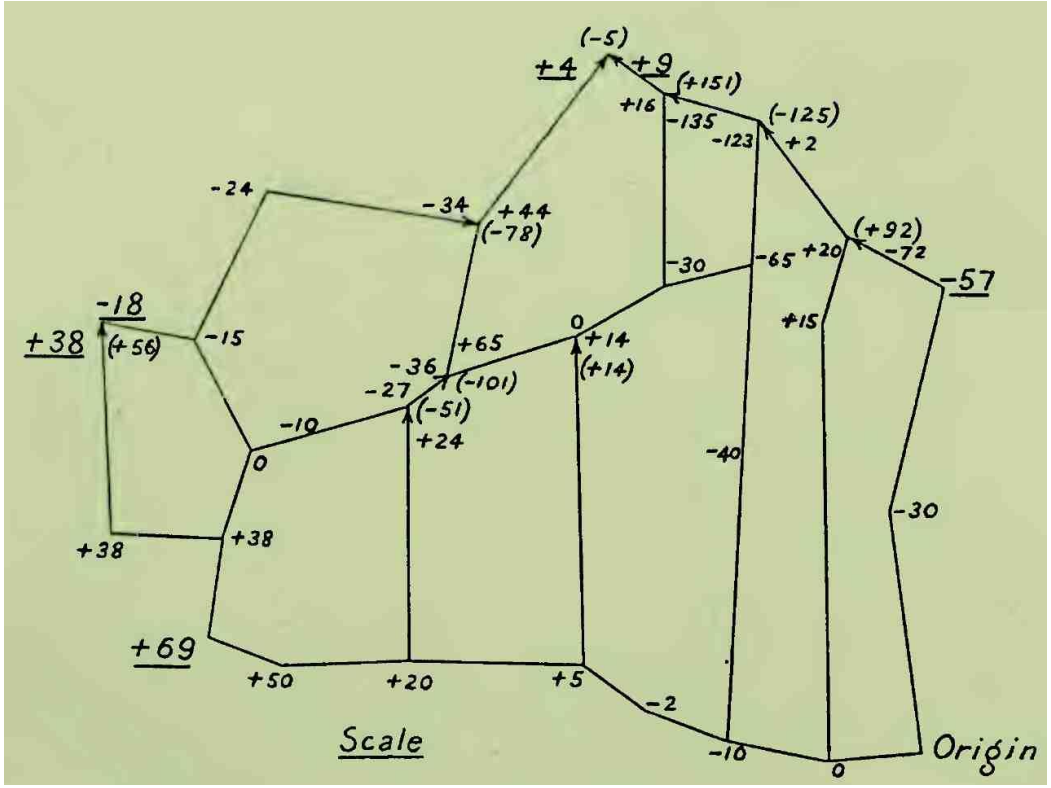


Indian Method of 1938

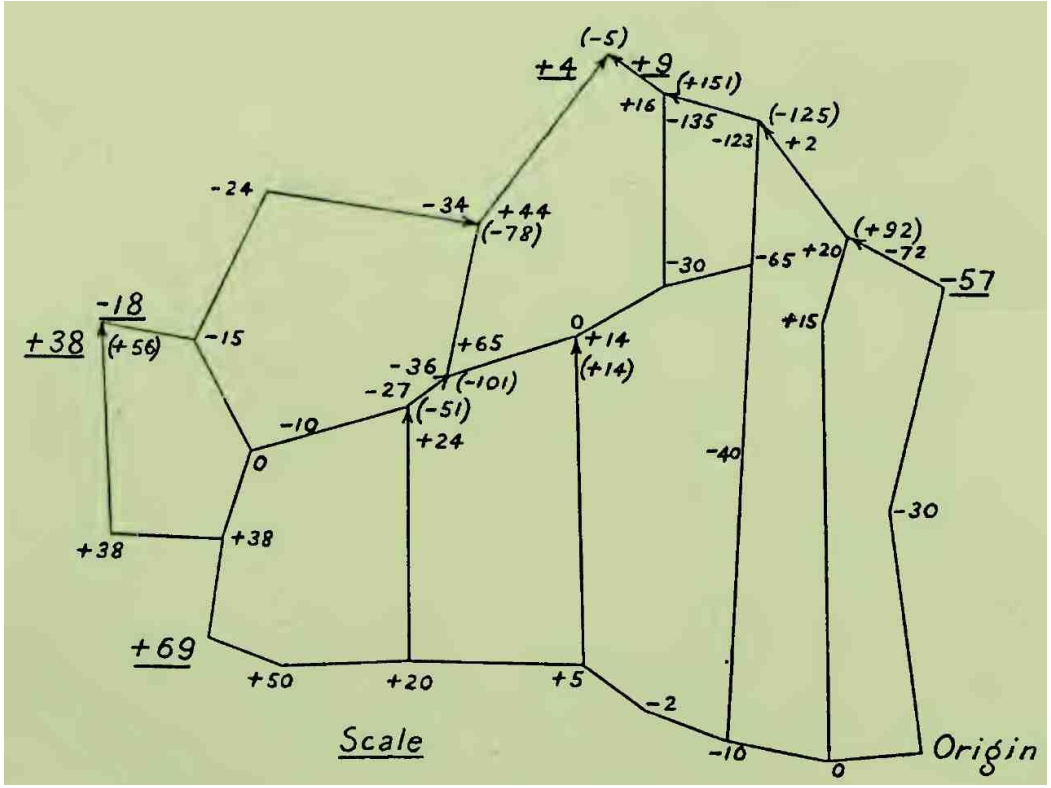


- Create a **spanning tree**
- Record errors in lat/long for other constraints
- Distribute errors in each loop

Indian Method of 1938



Indian Method of 1938

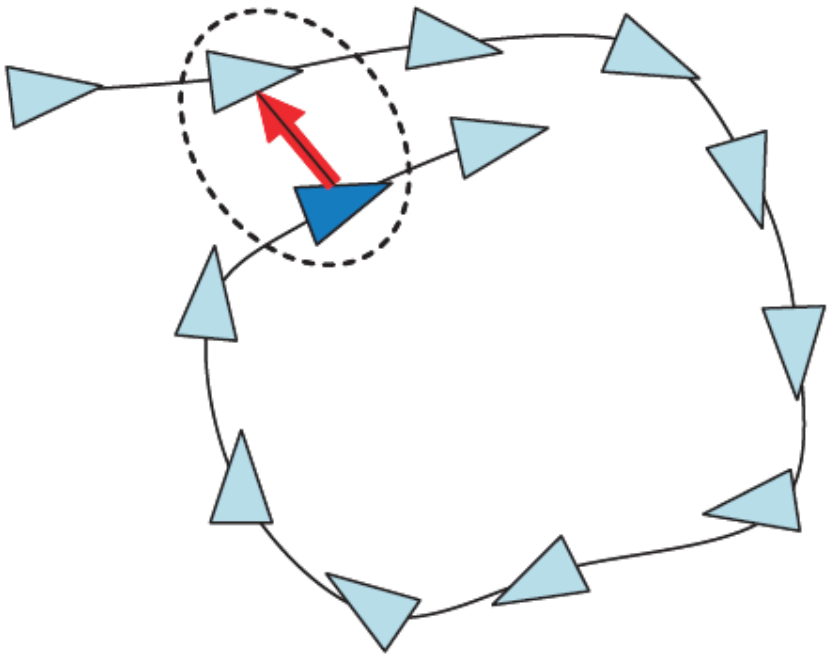


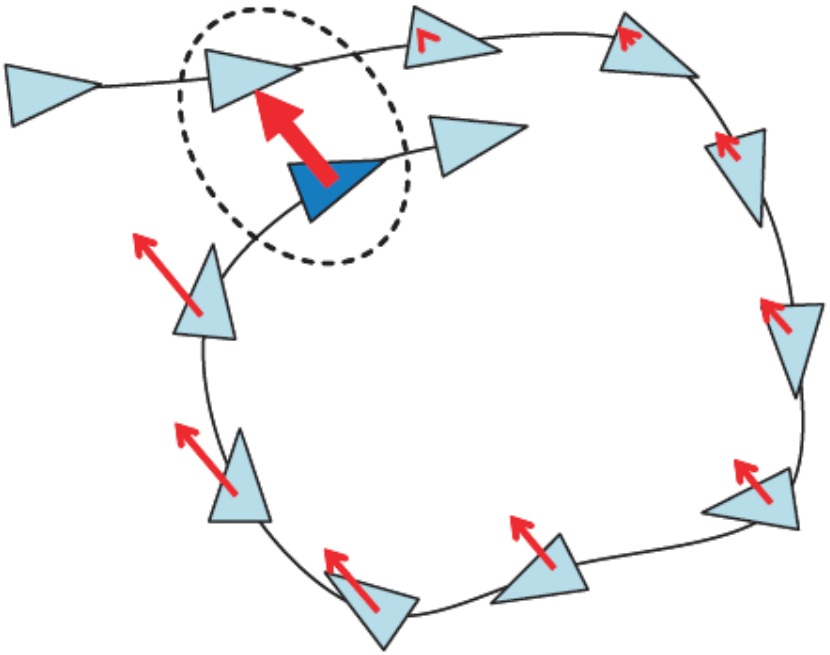
Behind a napkin!

Indian Method of 1938

Error distribution related to

Stochastic Gradient Descent, ICRA'07
TORO, RSS'07

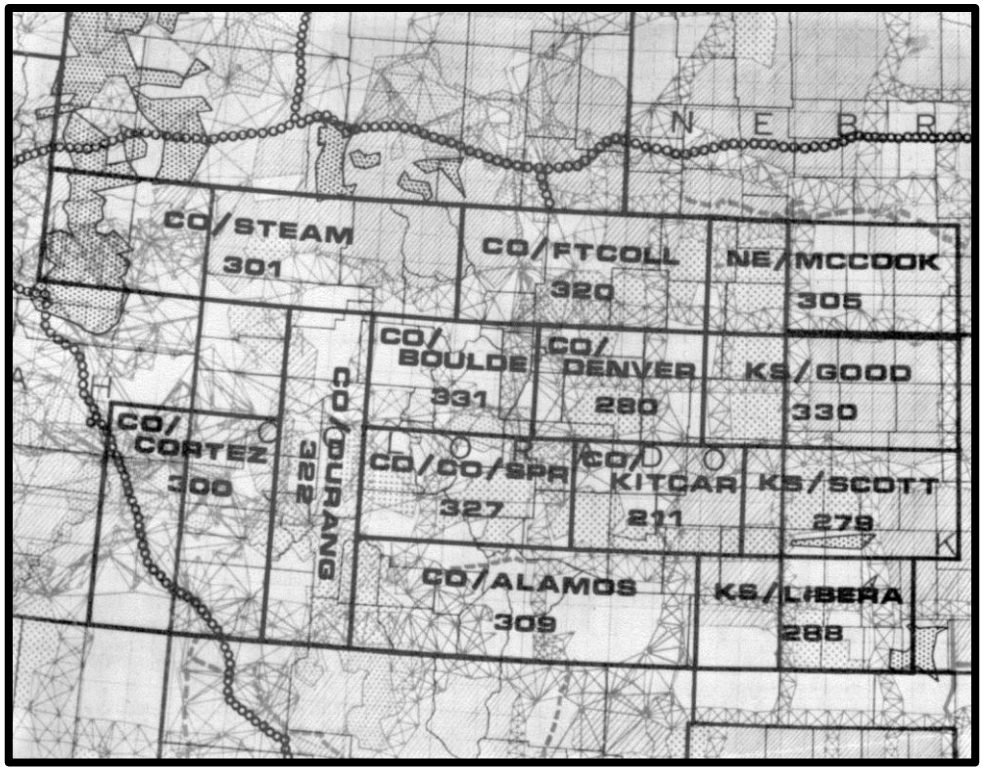




Images courtesy of Edwin Olson

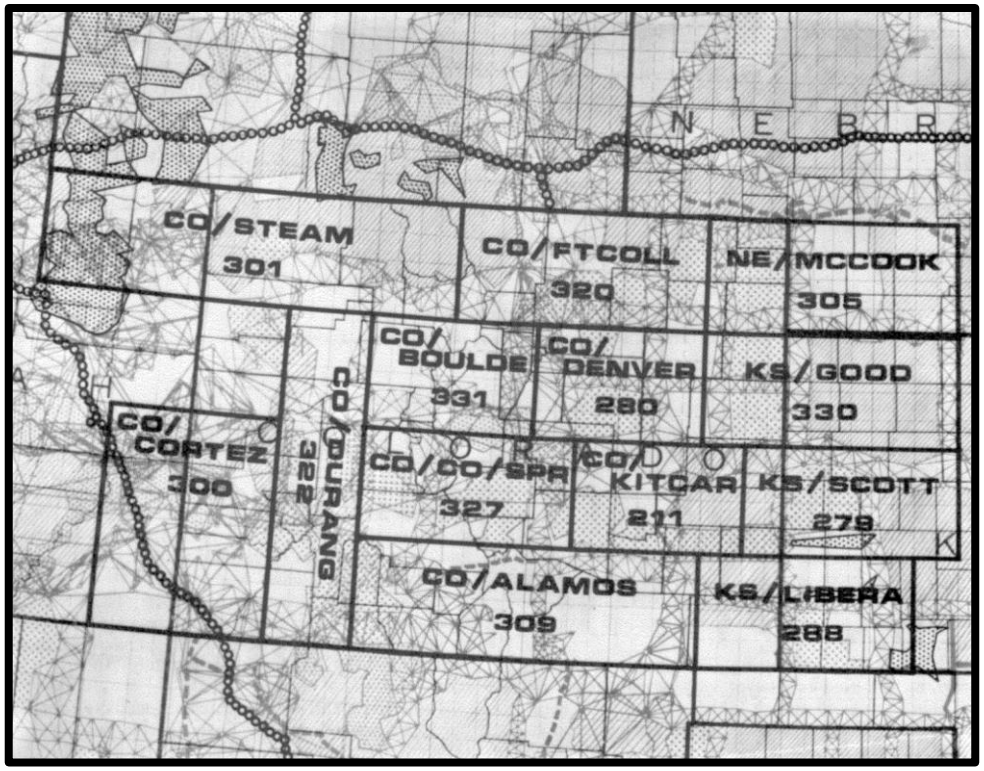
Handling Outliers

Inter and intra block consistency



Handling Outliers

Inter and intra block consistency



Related to **Robust loop closing over time, RSS'12**

Handling Outliers

- Remove observation if three other consistent ones exist

- Else keep doubling std-error till within 3σ

Handling Outliers

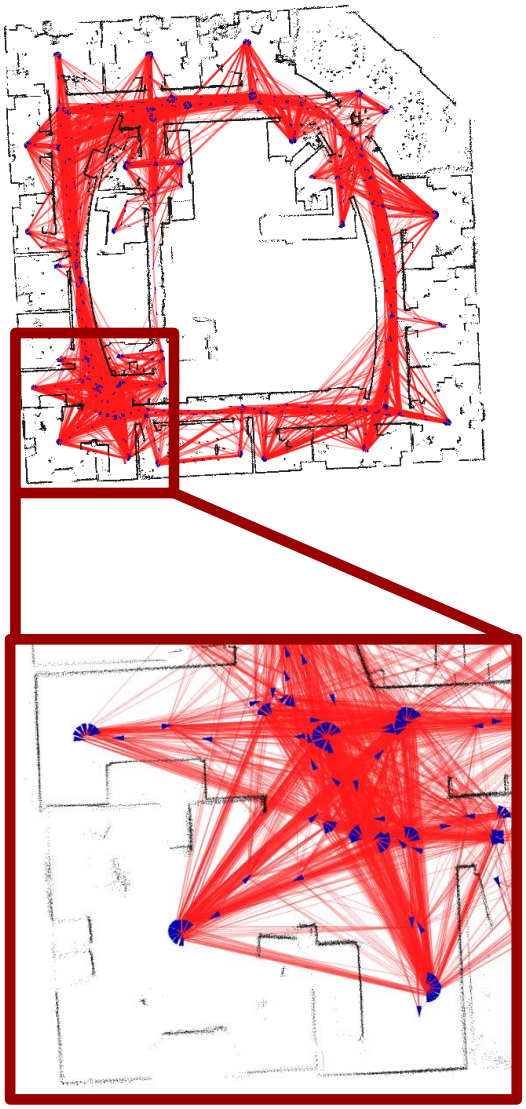
- Remove observation if three other consistent ones exist
- Else keep doubling std-error till within 3σ

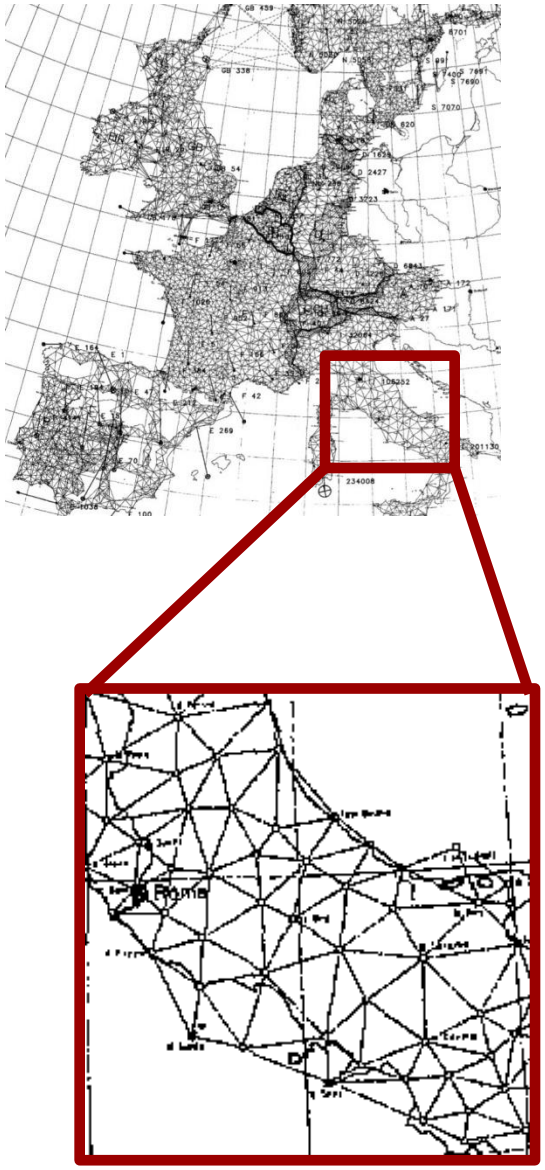
Related to **Robust M-estimators and DCS**

Additional Challenges for SLAM

- Autonomous vs. trained engineers
- Quality of initial guess
- Non-planarity of graphs

Non-Planarity of SLAM Graphs





Conclusion

- Survey of core geodetic mapping methods
- Share similarities

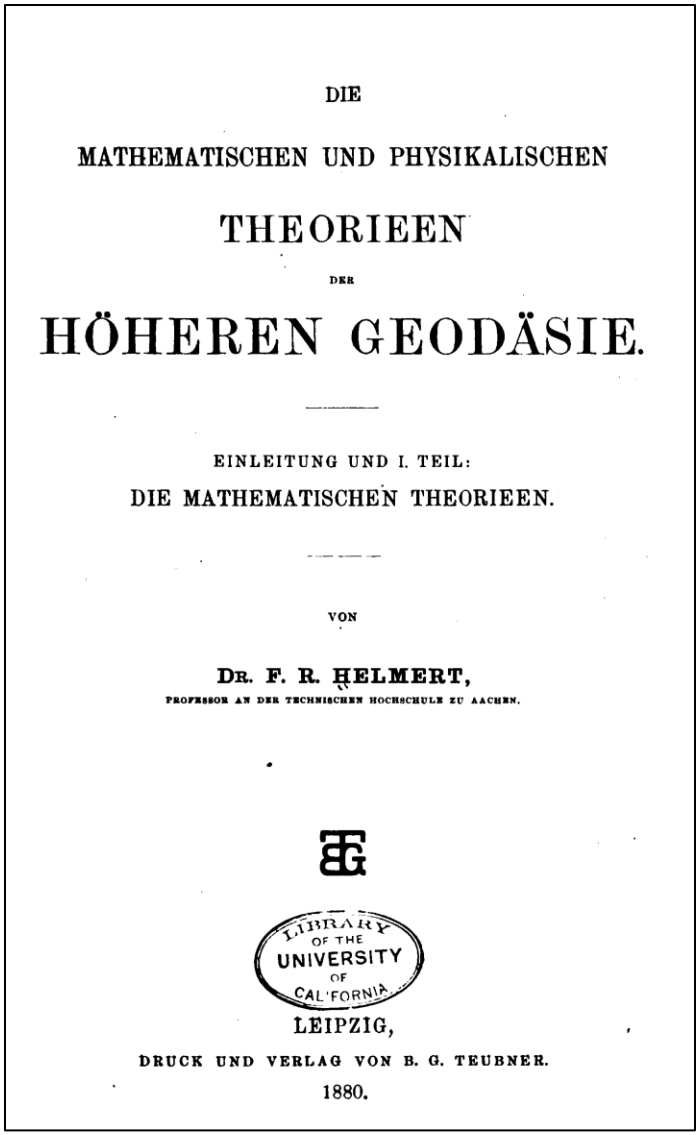
  Large maps

  Limited resources

  Outliers

  Non-linear observations
- Connect both fields for future synergies

Thank you for your attention!



DIE

MATHEMATISCHEN UND PHYSIKALISCHEN

THEORIEEN

DER

HÖHEREN GEODÄSIE.

EINLEITUNG UND I. TEIL:

DIE MATHEMATISCHEN THEORIEEN.

VON

DR. F. R. HELMERT,

PROFESSOR AN DER TECHNISCHEN HOCHSCHULE ZU AACHEN.

LEIPZIG,

DRUCK UND VERLAG VON B. G. TEUBNER.

1880.

Link to longer and detailed journal paper.