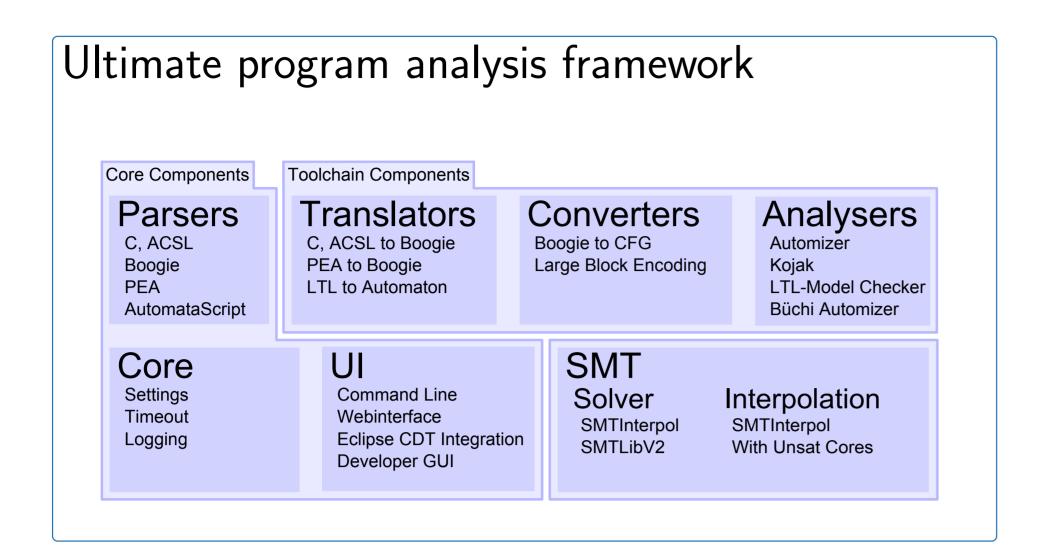
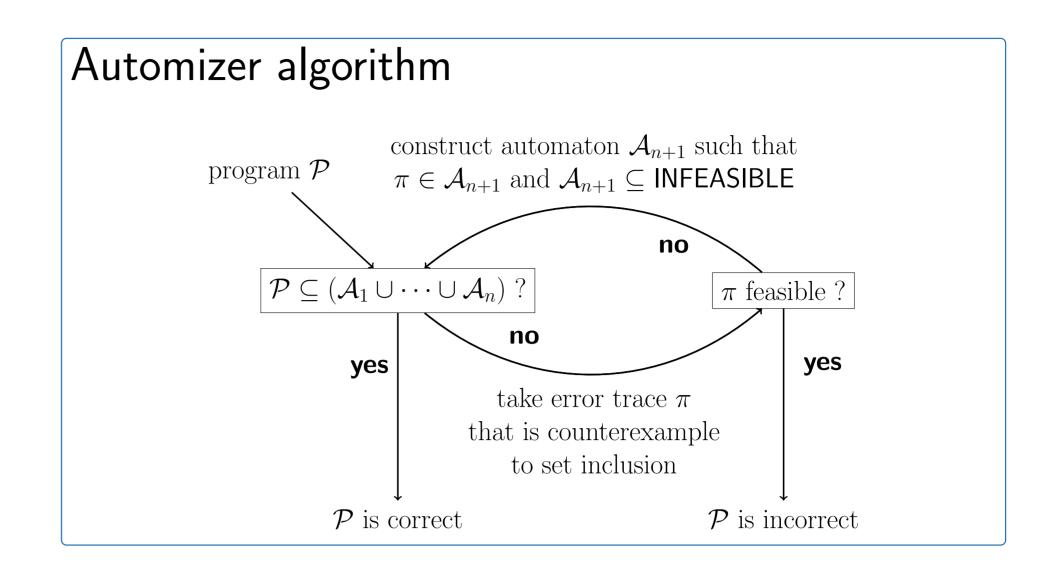
## ULTIMATE & FONTSON

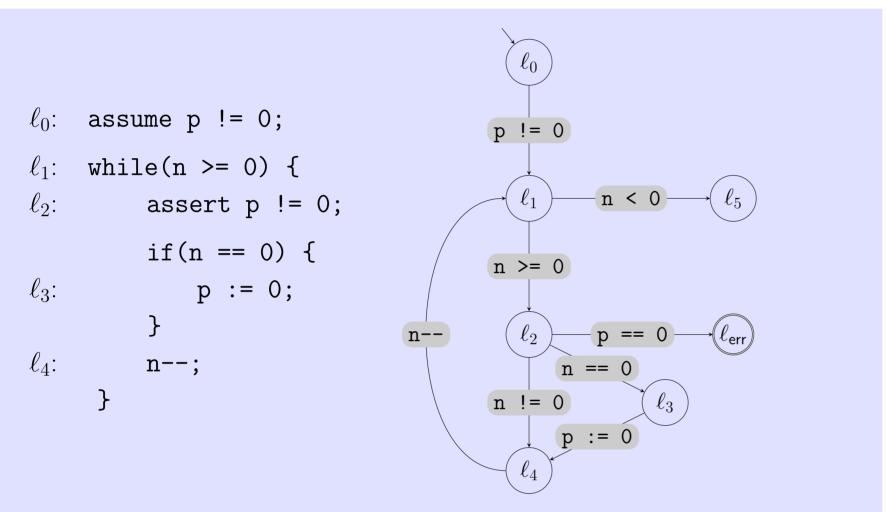
Matthias Heizmann, Jürgen Christ, Daniel Dietsch, Jochen Hoenicke, Markus Lindenmann, Betim Musa, Christian Schilling, Stefan Wissert, Andreas Podelski





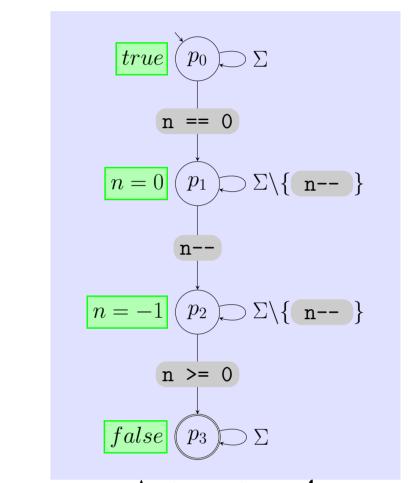
## Automata-theoretic proof of program correctness

Program  $\mathcal{P}$  is correct because each error trace is infeasible, i.e. the inclusion  $\mathcal{P} \subseteq \mathcal{A}_1 \cup \mathcal{A}_2$  holds.



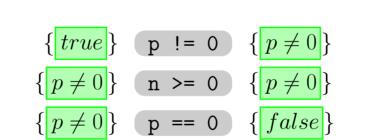
Program / automaton  $\mathcal{P}$  whose language is the set of error traces.

- p = 0  $p \neq 0 \quad q_1 \quad \Sigma \setminus \{ p := 0 \}$  p = 0  $false \quad q_2 \quad \Sigma$ 
  - Automaton  $A_1$  whose language is a set of infeasible traces.



Automaton  $A_2$ whose language is a set of infeasible traces.

- Alphabet: set of program statements  $\Sigma = \{ p := 0, n < 0, n >= 0, p == 0, n == 0, n != 0, p := 0, n -- \}$
- ullet The language of  ${\mathcal P}$  is the set of error traces.
- In the first iteration, we analyze feasibility of the error trace  $\pi_1 = p = 0$  n >= 0 p == 0.  $\pi_1$  is infeasible. Via interpolation, we obtain the following Hoare triples.



We construct the automaton  $\mathcal{A}_1$  such that its language is the set of all traces whose infeasibility can be shown using the predicates true,  $p \neq 0$ , and false.

- $\bullet$  Analogously, in the second iteration the automaton  $\mathcal{A}_2$  is constructed.
- We check the inclusion  $\mathcal{P} \subseteq \mathcal{A}_1 \cup \mathcal{A}_2$  and conclude that each error trace is infeasible and hence  $\mathcal{P}$  is correct.

**Definition** Given an automaton  $\mathcal{A} = (Q, \delta, q_{\mathsf{init}}, Q_{\mathsf{final}})$  over the alphabet of program statements, we call a mapping that assigns to each state  $q \in Q$  a predicate  $\varphi_q$  a Floyd-Hoare annotation for automaton  $\mathcal{A}$  if the following implications hold.

$$(q, s, q') \in \delta \implies \{\varphi_q\} s \{\varphi_{q'}\}$$
 is a valid Hoare triple  $q = q_{\mathsf{init}} \implies \varphi_q = true$   $q \in Q_{\mathsf{final}} \implies \varphi_q = false$ 

**Theorem** If an automaton  $\mathcal{A}$  has a Floyd-Hoare annotation, then  $\mathcal{A}$  recognizes a set of infeasible traces.

## Interpolation with unsatisfiable cores

Level 1: "interpolation" via

true

y = 0

 $y = 0 \land i = 0$ 

 $y = 0 \land i = 0 \land x = y$ 

 $y = 0 \land i = 1 \land x = y$ 

i := 0

x := y

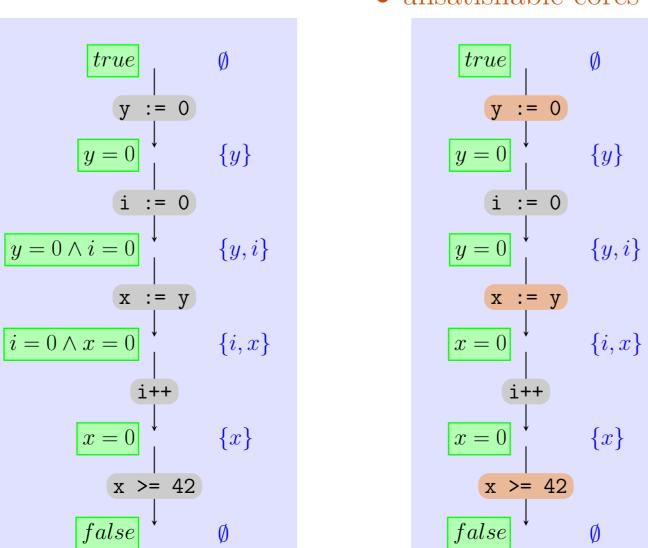
x >= 42

false

• strongest post

Level 2: interpolation via

- strongest post
- live variable analysis
- Level 3: interpolation via
- strongest post
- live variable analysis
- unsatisfiable cores



Algorithm (for level 3)

- Input: infeasible trace  $x_1, \ldots, x_n$  and unsatisfiable core  $\mathsf{UC} \subseteq \{x_1, \ldots, x_n\}$
- Replace each statement that does not occur in UC by a skip statement or a havoc statement.
  - assume statement  $\psi \rightsquigarrow \text{skip}$  assignment statement  $x:=t \rightsquigarrow \text{havoc } x$
- Compute sequence of predicates  $\varphi_0, \ldots, \varphi_n$  iteratively using the strongest post predicate transformer. sp

$$\varphi_0 := true$$
  
$$\varphi_{i+1} := sp(\varphi_i, \pounds_{i+1})$$

- Eliminate each variable from predicate  $\varphi_i$  that is not live at position i of the trace.
- Output: sequence of predicates  $\varphi_0, \ldots, \varphi_n$  which is a sequence of interpolants for the infeasible trace  $\mathfrak{x}_1, \ldots, \mathfrak{x}_n$