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Tutorial for Cyber-Physical Systems - Discrete Models

Exercise Sheet 1

General comments on our exercises

- First, try to understand the problem on your own, then discuss the problem (resp. your solution) at the university with your fellow students, finally write up the solution at home by yourself!

  The ability to discuss exercises/solutions is an important skill that can only be learned by doing it. Use this opportunity! However, working in a group can be very illusive. You might overlook several difficulties. In the exam you have to write down the solution by yourself. Use the exercises to prepare yourself for this situation as well!

- The mathematical background of this course's participants is very heterogeneous. Don't get frustrated if fellow students solve exercises much quicker than you do. We will have special exercises (e.g., Exercise 1 of this sheet) that help you to refresh math skills that are necessary for this lecture.

Exercise 1: Powerset — 3 Points

(a) Provide the powerset of

- the set $\{A, B\}$, and
- the set $\{A, B, C\}$.

(b) Prove the following claim via mathematical induction over the natural numbers.

The powerset of a set S has $2^{|S|}$ elements[1].

Hint: Compare the sets containing C to those not containing C in (a).

[1] Given a set S, we use $|S|$ to denote the number of elements in S.

Exercise 2*: Subset encoding 1 Point

Given a set S, provide a bit encoding of subsets S' of S (i.e., $S' \subseteq S$) with the following two properties:

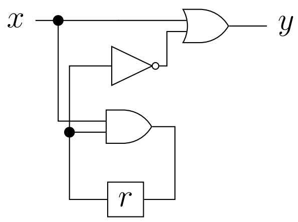
- There is a unique encoding for every subset of S (obviously).
- The encoding is optimal (i.e., which needs the minimal amount of bits).

Argue why your encoding has these properties.[2]

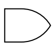
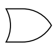
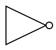
Hint: Use Exercise 1.

Exercise 3: Hardware circuit & transition system 2 Points

Consider the following sequential hardware circuit.



Provide the transition system of the hardware circuit analogously to the lecture. That is, the states are the evaluations of the inputs and the registers, the transitions represent the stepwise behavior where the values of the input bits change nondeterministically, and the atomic propositions are all input variables, registers and output variables whose value is one.
You may assume that initially the register r has the value `false`.

For your reference:  = AND gate,  = OR gate,  = NOT gate

[2] Note the star (*) next to the exercise number. It indicates a bonus exercise.