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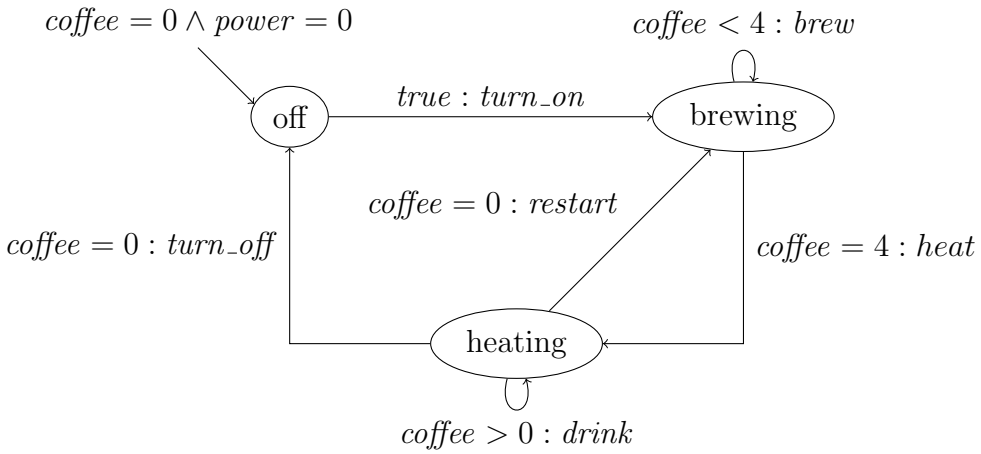
Tutorial for Cyber-Physical Systems - Discrete Models

Exercise Sheet 2

Exercise 1: Coffee Machine 3 Points

The following program graph describes a simple coffee machine:



The effect of the operations is given by:

$$\begin{aligned}
\mathit{Effect}(\mathit{turn_on}, \eta) &= \eta[\mathit{power} := 1] \\
\mathit{Effect}(\mathit{turn_off}, \eta) &= \eta[\mathit{power} := 0] \\
\mathit{Effect}(\mathit{brew}, \eta) &= \eta[\mathit{coffee} := \mathit{coffee} + 1] \\
\mathit{Effect}(\mathit{drink}, \eta) &= \eta[\mathit{coffee} := \mathit{coffee} - 1] \\
\mathit{Effect}(\mathit{restart}, \eta) &= \eta \\
\mathit{Effect}(\mathit{heat}, \eta) &= \eta
\end{aligned}$$

(a) Draw the transition system corresponding to the program graph.

(b) Check the following properties. Label the transition system with the corresponding atomic propositions given in parentheses.

(i) If the machine is turned off ($\mathit{power} = 0$) it contains no coffee ($\mathit{coffee} = 0$).

(ii) If there are two cups of coffee ($\mathit{coffee} = 2$) there are either three or four cups of coffee in the next step ($\mathit{coffee} = 3$, $\mathit{coffee} = 4$).

(iii) There are always at most four cups of coffee ($\mathit{coffee} \leq 4$).

(iv) The coffee machine will be eventually turned off.

(v) If there is no coffee ($\mathit{coffee} = 0$), there will be coffee after at most three steps.

Exercise 2*: Guarded command language I 2.5 Points

Consider the following program in guarded command language over the variables `x`, `y` whose domains are the integers $\mathbb{Z}$.

```
x := x + y;
y := x - y;
x := x - y
```

(a) List all statements of this program, including all substatements.

(b) Construct the corresponding program graph according to the method presented in the lecture, where we have a location for each statement. The set of initial locations Loc_0 is the singleton set that contains the statement that corresponds to the whole program.

(c) Which locations are reachable from the initial location?

(d) What is the relation between the value of the variables at the beginning and the value of the variables at the end?

(Note that we turned this exercise into a bonus exercise since the necessary background was not completely made clear in the lecture.)

Exercise 3: Guarded command language II 2 Points

Consider the following program in guarded command language over the variables `x`, `y` whose domains are the integers $\mathbb{Z}$.

```
DO ::  x > y ⇒ IF ::  x - y <= 5 ⇒ y := y - 1
                  ::  x <= 0     ⇒ x := x - 1
               FI
OD
```

(a) Construct the corresponding program graph. (You do not have to use the method presented in the lecture.)

(b) We say that a program is *terminating* if the corresponding transition system of its program graph does not have an infinite execution.

Is the above program terminating? An informal argument is sufficient.