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# Tutorial for Cyber-Physical Systems - Discrete Models

## Exercise Sheet 5

**Exercise 1: Regular expressions and $\omega$-regular expressions** 4 Points

We use the convention that we omit parentheses in expressions if the meaning is clear.

(a) Consider the alphabet $\Sigma = \{a, b\}$. Decide for each of the following strings whether it describes a regular expression or an $\omega$-regular expression. If so, describe the language.

(i) $\emptyset a \emptyset$

(ii) $a^* b$

(iii) $\varepsilon^{**}$

(iv) $a^{\omega\omega}$

(v) $\emptyset^{*\omega}$

(vi) $a(ba)^\omega + (ab)^\omega$

(b) Construct regular expressions resp. $\omega$-regular expressions for the following languages over the alphabet $\Sigma = \{a, b\}$.

(i) $L_1 = \{w \in \Sigma^* \mid \text{every } a \text{ in } w \text{ is immediately followed by } b\}$

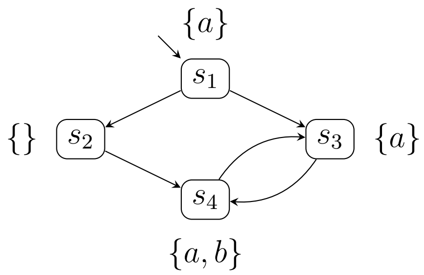
(ii) $L_2 = \{w \in \Sigma^* \mid w \text{ does not contain } bb\}$

(iii) $L_3 = \{w \in \Sigma^\omega \mid w \text{ contains at least two } a\}$

(iv) $L_4 = \{w \in \Sigma^\omega \mid w \text{ contains at most two } a\}$

**Exercise 2: Traces** 1 Point

Consider the following transition system with atomic propositions $AP = \{a, b\}$.




Describe the traces of the transition system using an $\omega$-regular expression over the alphabet $\Sigma = 2^{AP}$.

**Exercise 3: LT properties** 3 Points

Assume $AP = \{a, b\}$. Define the following properties as a set of traces.

(a) Every state satisfies $a$ or $b$.

(b) There is no state satisfying $b$ before the first occurence of $a$.

(c) Every $a$ will be eventually followed by $b$.

(d) Exactly three states satisfy $a$.

(e) If there are infinitely many $a$, then there are infinitely many $b$.

(f) There are only finitely many $a$.

*Note*: If "state" is mentioned in the property, we mean a state in an execution of the transition system, not an arbitrary (possibly unreachable) state of the transition system. For example, the three states in the fourth property may also be the same state visited three times.

**Exercise 4*: Terminal states** 2 Points

In the lecture we described a transformation of a transition system with possible terminal states into an "equivalent" transition system without terminal states. Note that the transformation preserves trace-equivalence, i.e., if $TS_1$, $TS_2$ are transition systems (possibly with terminal states) such that $Traces_{\text{fin}}(TS_1) = Traces_{\text{fin}}(TS_2)$, and $TS'_1$, $TS'_2$ are the transformations of $TS_1$, $TS_2$, respectively, then $Traces(TS'_1) = Traces(TS'_2)$.

Give a formal definition of this transformation.

*Hint*: You may describe the transformation as follows.
Given a transition system $TS = (S, Act, \rightarrow, I, AP, L)$, describe the construction of the corresponding transition system $TS' = (S', Act', \rightarrow', I', AP', L')$.

*Note*: This exercise became a bonus exercise. The original exercise contained an error.