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Discussion: May 14, 2018

Tutorial for Program Verification
Exercise Sheet 3 – Part 1/2

Exercise 1: Formalization in first-order logic 2 Points

Express the following declarative sentences in first-order logic; in each case state what your respective constant, function, and predicate symbols stand for:

(a) Whatever goes upon four legs, or has wings, is a friend.

(b) No animal shall kill any other animal.

(c) All animals are equal, but some animals are more equal than others.

(d) The array a, whose indices and values are integers, is sorted between position 0 and position l.

Exercise 2: Quantifiers 2 Points

(a) Show that the following first-order logic formula is not valid.

$$((\forall x.\ P(x)) \to Q) \ \to\ (\forall x.\ P(x) \to Q)$$

(b) Is the other direction of the implication (s. below) valid?

$$(\forall x.\ P(x) \to Q) \ \to\ ((\forall x.\ P(x)) \to Q)$$

A short argument is sufficient.

Exercise 3: Minimal unsatisfiable core 2 Points

Definition (Minimal unsatisfiable core) Let Γ be a finite set of formulas such that the conjunction $\bigwedge_{\phi\in\Gamma} \phi$ is unsatisfiable. A subset $\Gamma' \subseteq \Gamma$ is called *unsatisfiable core* of Γ if $\bigwedge_{\phi\in\Gamma'} \phi$ is also unsatisfiable. An unsatisfiable core Γ' is called *minimal unsatisfiable core* if for each proper subset $\Gamma'' \subsetneq \Gamma'$ the conjunction $\bigwedge_{\phi\in\Gamma''} \phi$ is satisfiable.

(a) Give a minimal unsatisfiable core for the following set of formulas.

$$\{ \quad \neg(X \to \neg Z), \quad Y \to \neg U, \quad X \to Y, \quad X, \quad Z \to U \quad \}$$

(b) Is the minimal unsatisfiable core of a set of formulas unique? (Are there sets of formulas $\Gamma, \Gamma_1, \Gamma_2$ such that $\Gamma_1 \neq \Gamma_2$ but both Γ_1 and Γ_2 are minimal unsatisfiable cores of Γ?)