Tutorial for Program Verification
Exercise Sheet 3 – Part 1/2

Exercise 1: Formalization in first-order logic 2 Points
Express the following declarative sentences in first-order logic; in each case state what your respective constant, function, and predicate symbols stand for:

(a) Whatever goes upon four legs, or has wings, is a friend.

(b) No animal shall kill any other animal.

(c) All animals are equal, but some animals are more equal than others.

(d) The array $a$, whose indices and values are integers, is sorted between position 0 and position $l$.

Exercise 2: Quantifiers 2 Points
(a) Show that the following first-order logic formula is not valid.

$$((\forall x. P(x)) \rightarrow Q) \rightarrow (\forall x. P(x) \rightarrow Q)$$

(b) Is the other direction of the implication (s. below) valid?

$$(\forall x. P(x) \rightarrow Q) \rightarrow ((\forall x. P(x)) \rightarrow Q)$$

A short argument is sufficient.

Exercise 3: Minimal unsatisfiable core 2 Points

| Definition (Minimal unsatisfiable core) | Let $\Gamma$ be a finite set of formulas such that the conjunction $\bigwedge_{\phi \in \Gamma} \phi$ is unsatisfiable. A subset $\Gamma' \subseteq \Gamma$ is called unsatisfiable core of $\Gamma$ if $\bigwedge_{\phi \in \Gamma'} \phi$ is also unsatisfiable. An unsatisfiable core $\Gamma'$ is called minimal unsatisfiable core if for each proper subset $\Gamma'' \subsetneq \Gamma'$ the conjunction $\bigwedge_{\phi \in \Gamma''} \phi$ is satisfiable. |

(a) Give a minimal unsatisfiable core for the following set of formulas.

$$\{ \neg(X \rightarrow \neg Z), \ Y \rightarrow \neg U, \ X \rightarrow Y, \ X, \ Z \rightarrow U \}$$

(b) Is the minimal unsatisfiable core of a set of formulas unique? (Are there sets of formulas $\Gamma, \Gamma_1, \Gamma_2$ such that $\Gamma_1 \neq \Gamma_2$ but both $\Gamma_1$ and $\Gamma_2$ are minimal unsatisfiable cores of $\Gamma$?)