Exercise 1: Hoare logic

In this exercise we consider very simple Hoare triples over Boolean variables where

- the precondition $\text{precond}(X_1, \ldots, X_n)$ is a Boolean expression over the Boolean variables $X_1, \ldots, X_n$ and does not contain the Boolean variable $Y$,
- the program consists of the single line

$$Y := \text{expr}(X_1, \ldots, X_n),$$

where $Y$ is a Boolean variable and $\text{expr}(X_1, \ldots, X_n)$ is a Boolean expression over the Boolean variables $X_1, \ldots, X_n$ that does not contain $Y$, and
- the postcondition $\text{postcond}(X_1, \ldots, X_n)$ is a Boolean expression over the variables $Y, X_1, \ldots, X_n$.

(a) State a propositional logical formula

$$vc(Y, X_1, \ldots, X_n)$$

that is valid if and only if a Hoare triple that has the following form is valid.

$$\{ \text{precond}(X_1, \ldots, X_n) \} \ Y := \text{expr}(X_1, \ldots, X_n) \{ \text{postcond}(Y, X_1, \ldots, X_n) \}$$

(b) Compute your propositional logical formula $vc(Z, U, V)$ for the following concrete program.

$$\{ \ U \leftrightarrow V \ \} \ Z := U \land V \ \{ \ Z \leftrightarrow U \ \}$$

Is your formula valid?

(c) Now we drop the restriction that $\text{precond}(X_1, \ldots, X_n)$ does not contain the Boolean variable $Y$. Find a Hoare triple that is not valid but where your formula $vc(U, V, Z)$ is valid.
Exercise 2: Hoare logic derivation

(a) Write down a partial correctness specification (i.e., precondition and postcondition) for a program $C$ that computes the maximum of $x$ and $y$ and stores the result in $z$.

(b) Write down the program $C$. Use the syntax for programs introduced in the lecture.

(c) Construct a Hoare logic derivation that proves that your program $C$ fulfills your correctness specification.

Exercise 3: Hoare triples

Consider the following Hoare triples. Which of them are valid for any program $C$ and any state assertion $\phi$?

(a) \{ true \} $C$ \{ $\phi$ \}

(b) \{ false \} $C$ \{ $\phi$ \}

(c) \{ $\phi$ \} $C$ \{ true \}

(d) \{ $\phi$ \} $C$ \{ false \}

If a Hoare triple is valid for any program $C$ and any state assertion $\phi$, then explain why. If a Hoare triple is not valid for some program $C$ and some state assertion $\phi$, then give a counterexample.