Exercise 1: Hoare logic

In this exercise we consider very simple Hoare triples over Boolean variables where

- the precondition \( precond(X_1, \ldots, X_n) \) is a Boolean expression over the Boolean variables \( X_1, \ldots, X_n \) and does not contain the Boolean variable \( Y \),
- the program consists of the single line
  
  \[ Y := \text{expr}(X_1, \ldots, X_n), \]

  where \( Y \) is a Boolean variable and \( \text{expr}(X_1, \ldots, X_n) \) is a Boolean expression over the Boolean variables \( X_1, \ldots, X_n \) that does not contain \( Y \), and
- the postcondition \( postcond(X_1, \ldots, X_n) \) is a Boolean expression over the variables \( Y, X_1, \ldots, X_n \).

(a) State a propositional logical formula

\[ vc(Y, X_1, \ldots, X_n) \]

that is valid if and only if a Hoare triple that has the following form is valid.

\[ \{ \ precond(X_1, \ldots, X_n) \ \} \ \ Y := \text{expr}(X_1, \ldots, X_n) \ \{ \ postcond(Y, X_1, \ldots, X_n) \ \} \]

(b) Compute your propositional logical formula \( vc(Z, U, V) \) for the following concrete program.

\[ \{ \ U \leftrightarrow V \ \} Z := U \land V \{ \ Z \leftrightarrow U \ \} \]

Is your formula valid?

(c) Now we drop the restriction that \( precond(X_1, \ldots, X_n) \) does not contain the Boolean variable \( Y \). Find a Hoare triple that is not valid but where your formula \( vc(U, V, Z) \) is valid.
Exercise 2: Hoare logic derivation 2 Points

(a) Write down a partial correctness specification (i.e., precondition and postcondition) for a program C that computes the maximum of x and y and stores the result in z.

(b) Write down the program C. Use the syntax for programs introduced in the lecture.

(c) Construct a Hoare logic derivation that proves that your program C fulfills your correctness specification.

Exercise 3: Hoare triples 2 Points

Consider the following Hoare triples. Which of them are valid for any program C and any state assertion φ?

(a) { true } C { φ }

(b) { false } C { φ }

(c) { φ } C { true }

(d) { φ } C { false }

If a Hoare triple is valid for any program C and any state assertion φ, then explain why. If a Hoare triple is not valid for some program C and some state assertion φ, then give a counterexample.