Tutorial for Program Verification
Exercise Sheet 9

Exercise 1: Soundness of the Hoare Proof System 0 Points

How can we prove the following Theorem of the lecture?

Theorem (Soundness of the Hoare proof system) If there is a derivation whose root is
labelled by the Hoare triple \( \{ \varphi \} st \{ \psi \} \) then the statement \( st \) satisfies the precondition-postcondition pair \( \{(\varphi), (\psi)\}\)

On Wednesday, we will discuss your ideas and carry out some parts of the proof in the
lecture. Carrying out the remaining parts of the proof will become an exercise of Exercise
Sheet 10.

Exercise 2: While Rule of the Hoare Proof System 2 Points

Use the Hoare proof system to show that the Hoare triple

\[
\{ \begin{array}{c}
  b \leq -23 \\
  \land a \geq 42
\end{array} \} \quad \text{while}(\neg(b==0)) \{ \begin{array}{c}
    \text{if}(b>=0) \{ b:=b-1; \} \\
    \text{else} \{ b:=b+1; \}
\end{array}\} \{ \begin{array}{c}
    a:=a+1; \\
  \end{array}\} \quad \{ a \geq 47 \}
\]

is valid. You may use that the following Hoare triple is valid. (I.e., you may use this
Hoare triple as a leaf of the derivation tree.)

\[
\{ \begin{array}{c}
  (b\leq 0 \rightarrow a-b\geq 53) \\
  \lor (a\geq 37 \land b\leq -17)
\end{array} \} \quad \text{if}(b>=0) \{ b:=b-1; \} \{ b:=b+1; \} \{ a:=a+1; \} \quad \{ b \leq 0 \rightarrow a - b \geq 53 \}
\]

Hint: The while rule is not sufficient, you also need the rules (strepre) and (weakpos).

The challenge is to find an inductive loop invariant \( \varphi \) that is implied by the precondition
of the first Hoare triple above and that implies the precondition of the second Hoare
triple above.