In Lecture 8 we made the following definition.

**Definition (Post Image)** Given a binary relation $R$ over the set $X$ and a subset of $Y \subseteq X$, the *post image of $Y$ under $R$*, denoted $post(Y, R)$, is the set $\{x \in X \mid \text{exists } y \in Y \text{ such that } (y, x) \in R\}$

We use the post image to give a formal definition of the *strongest postcondition* for a given set of program states $S$ and a given statement $st$. Intuitively, the strongest postcondition is the set of states in which a program can be after executing $st$ in some state $s \in S$.

**Definition (Strongest Postcondition)** Given a set of states $S$ and a statement $st$ the *strongest postcondition* is the post image of $S$ under the relation $\llbracket st \rrbracket$, i.e.

$$sp(S, st) = post(S, \llbracket st \rrbracket).$$

**Exercise 1: Strongest Postcondition**

Below, you find six sets of states that are each given as a strongest postcondition. Write down each set without using the strongest postcondition operator. You may use any formalism that you have seen in the lecture. Recall that $\{\varphi\}$ denotes the set of states that satisfy the formula $\varphi$. In the formulas below, $i, k, x$ are integer variables and $a$ is an array whose indices and values are integers.

(a) $sp(\{select(a, k) = 23 \land select(a, i) = 42\}, \text{assume } i=k; )$

(b) $sp(\{0 \leq k \land k \leq i\}, \text{havoc } k; )$

(c) $sp(\{select(a, 23) = 42\}, a[k]:=1337; )$

(d) $sp(\{x \cdot x > 5\}, x:=k-i; )$

(e) $sp(\{x\%2 = 0\}, x:=x+1; )$

(f) $sp(\{select(a, i + 1) = 23\}, i:=2*2+k; )$