Exercise 1: Strongest Postcondition for the Conditional Statement  
2 Points

In the lecture we have seen that the strongest postcondition of a sequential composition or a while-loop can be expressed in terms of the strongest postconditions of the sub-statements. In this exercise, you should give a similar formulation for the strongest postcondition of an conditional statement.

Specifically, let $st$ be the statement $\text{if (expr) \{ st_1 \} else \{ st_2 \}}$. Let $S \subseteq S_{V}^{\mu}$ be a set of states. Express the strongest postcondition $sp(S, st)$ using only the strongest postcondition of $st_1$, $st_2$ and of simple statements (havoc, assignments or assume) for suitable sets of states.

You do not have to prove the correctness of your result.

Exercise 2: Distributivity of $sp$  
4 Points

In this exercise we examine distributivity properties of the strongest postcondition. Let $S, S_1, S_2$ be arbitrary sets of states, and let $st$ be a statement. Furthermore, let $\varphi_1$ and $\varphi_2$ be formulas.

For each of the following equalities, either prove its correctness or give a counterexample.

(a) $sp(S_1 \cup S_2, st) = sp(S_1, st) \cup sp(S_2, st)$

(b) $sp(S_1 \cap S_2, st) = sp(S_1, st) \cap sp(S_2, st)$

(c) $sp(S, \text{assume } \varphi_1 \lor \varphi_2) = sp(S, \text{assume } \varphi_1) \cup sp(S, \text{assume } \varphi_2)$

(d) $sp(S, \text{assume } \varphi_1 \land \varphi_2) = sp(S, \text{assume } \varphi_1) \cap sp(S, \text{assume } \varphi_2)$

Exercise 3: Strongest Postcondition  
2 Points

Consider the following program $P$.

```
1 assume x > y;
2 x := x - y;
3 havoc z;
4 assume z > 0;
5 x := x * z;
```

Compute the strongest postcondition $sp(S, P)$ where $S$ is $\{y > 0\}$. 

Exercise 4: Weakest Precondition

Analogously to the strongest postcondition we define the weakest precondition for a given set of states and a given statement \( st \) as follows.

\[
wp(S, st) = \{ s \in S_{V\mu} \mid \forall s' \in S_{V\mu}, \ (s, s') \in [st] \text{ implies } s' \in S \}
\]

Intuitively, the weakest precondition is the set of states such that if we can execute \( st \) and \( st \) terminates then we are in some state of \( S \).

Let us assume that the set \( S \) is given by a formula \( \psi \), i.e., \( S = \{ \psi \} \). Give a formula \( \varphi \) such that \( wp(S, st) = \{ \varphi \} \) for the cases where

(a) \( st \) is an assignment statement of the form \( x := \text{expr} \),

(b) \( st \) is an assume statement of the form \( \text{assume expr} \), and

(c) \( st \) is a havoc statement of the form \( \text{havoc x} \).