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Hand in until 16:15 on July 10, 2019
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Tutorial for Program Verification
Exercise Sheet 19

On this exercise sheet, we will work with complete lattices.

Exercise 1: Divisibility 0 Points

Consider the complete lattice $(L, |, \sqcap, \sqcup)$ with $L = \{1, 2, 3, 4, 6, 12\}$, where $|$ is the divisibility relation on integers.

(a) Compute $\bigsqcup L$ and $\bigsqcap L$.

(b) Compute $\bigsqcup\{3, 4, 6\}$ and $\bigsqcap\{4, 6, 12\}$.

(c) Why is $(\mathbb{Z}, |, \sqcap, \sqcup)$ not a complete lattice?

Exercise 2: Intervals 2 Points

Let $L = \{[a, b] \mid a, b \in \mathbb{Z} \cup \{-\infty, +\infty\}\}$ be the set of the closed intervals over the integers $\mathbb{Z}$ extended by $-\infty$ and $+\infty$. In this definition, $\pm\infty$ have the usual meaning, and as usual, $[a, b] = \varnothing$ for $a > b$.
Let the partial order $\sqsubseteq$ on L be given by the subset relation $\subseteq$.

Give the operator $\bigsqcup$ for the least upper bound and the operator $\bigsqcap$ for the greatest lower bound such that $(L, \sqsubseteq, \sqcap, \sqcup)$ is a complete lattice.