Off this exercise sheet, we will work with complete lattices.

Exercise 1: Divisibility 0 Points
Consider the complete lattice \((L, |, \sqcap, \sqcup)\) with \(L = \{1, 2, 3, 4, 6, 12\}\), where \(\mid\) is the divisibility relation on integers.

(a) Compute \(\sqcup\) and \(\sqcap\).

(b) Compute \(\sqcup\{3, 4, 6\}\) and \(\sqcap\{4, 6, 12\}\).

(c) Why is \((\mathbb{Z}, |, \sqcap, \sqcup)\) not a complete lattice?

Exercise 2: Intervals 2 Points
Let \(L = \{[a, b] | a, b \in \mathbb{Z} \cup \{-\infty, +\infty\}\}\) be the set of the closed intervals over the integers \(\mathbb{Z}\) extended by \(-\infty\) and \(+\infty\). In this definition, \(\pm\infty\) have the usual meaning, and as usual, \([a, b] = \emptyset\) for \(a > b\).

Let the partial order \(\subseteq\) on \(L\) be given by the subset relation \(\subseteq\).

Give the operator \(\sqcup\) for the least upper bound and the operator \(\sqcap\) for the greatest lower bound such that \((L, \subseteq, \sqcap, \sqcup)\) is a complete lattice.