Tutorial for Program Verification
Exercise Sheet 1

In this exercise we practice some of the key concepts of Propositional Logic (PL).
Submit your solution by uploading it as PDF in ILIAS.

Exercise 1: Satisfiability, Validity 2 Points
Is the following PL formula satisfiable? Is the following PL formula valid?
If the formula is satisfiable then give a variable assignment such that formula is evaluated
to true. If the formula is not valid then give a variable assignment such that formula is
evaluated to false.

\[ C \rightarrow (A \lor (B \land C)) \]

Exercise 2: Conjunctive Normal Form 3 Points
We call two PL formulas \( F_1 \) and \( F_2 \) logically equivalent, denoted \( F_1 \equiv F_2 \), if they evaluate
to the same truth value under every variable assignment.
We say that a formula \( F \) is in conjunctive normal form (CNF) if it is a conjunction of
disjunctions of literals, i.e., if it has the form

\[ \bigwedge_i \bigvee_j \ell_{ij} \]

where \( \ell_{ij} \) are literals.
Any formula can be transformed into an equivalent formula in CNF using the following
template equivalences (left to right):

\[
\begin{align*}
\neg\neg F_1 & \equiv F_1 \\
\neg(F_1 \land F_2) & \equiv \neg F_1 \lor \neg F_2 \\
\neg(F_1 \lor F_2) & \equiv \neg F_1 \land \neg F_2 \\
F_1 \rightarrow F_2 & \equiv \neg F_1 \lor F_2 \\
F_1 \leftrightarrow F_2 & \equiv (F_1 \rightarrow F_2) \land (F_2 \rightarrow F_1) \\
(F_1 \land F_2) \lor F_3 & \equiv (F_1 \lor F_3) \land (F_2 \lor F_3) \\
F_1 \lor (F_2 \land F_3) & \equiv (F_1 \lor F_2) \land (F_1 \lor F_3)
\end{align*}
\]

De Morgan’s Law

Transform the following formulas into an equivalent formula in CNF.

(a) \( A \land B \rightarrow A \lor B \)

(b) \( C \rightarrow (A \lor (B \land C)) \)
Exercise 3: The NOR Connective
3 Points
In the lecture we defined the syntax of propositional logic by using only false and the logical connectives ¬ and ∧. (The other logical connectives were introduced as abbreviations.) In this exercise we show that alternatively we could have defined the syntax of propositional logic by using only a single logical connective.
Given two PL formulas $F_1$ and $F_2$, we define the logical connective NOR ($\bar{\lor}$) by the following truth table:

<table>
<thead>
<tr>
<th>$F_1$</th>
<th>$F_2$</th>
<th>$F_1 \bar{\lor} F_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 1: Truth table for NOR

Show that the atom false and the logical connectives ¬ and ∧ can be expressed by the NOR connective $\bar{\lor}$, i.e., given arbitrary PL formulas $F, F_1, F_2$ state for each of the PL formulas false, $\neg F$, and $(F_1 \land F_2)$ a PL formula that is logically equivalent but uses only $F, F_1, F_2$, and $\bar{\lor}$.
In this exercise it is sufficient to state a formula without a proof of logical equivalence.

Exercise 4: Birthday Wishes
4 Points
Annika was always a little eccentric, but when she presented her family and friends with this year’s list of birthday wishes (copied below), they couldn’t believe their eyes. Can you help them? Encode the constraints in boolean formulae and find a satisfying assignment in order to find a combination of presents that satisfies Annika’s demands. You may use an SMT solver (e.g. Z3) to obtain the satisfying assignment.

To all my friends and family!
you asked me what I wished for on my birthday, so here’s my list:
If one of my presents is going to be a Netflix subscription, then I don’t want to receive the new Ed Sheeran album. If you are going to give me an iPhone XR, then I don’t want a pair of Adidas Yeezy Sneakers. However, if you give me the sixth A Song of Ice and Fire book, then I would like the Netflix subscription and tickets for Mark Forster.
If you do not get me Adidas Yeezy Sneakers as a present, then I want to receive the sixth A Song of Ice and Fire book or a selfie stick. If you do not bring me a selfie stick, then I ask you to bring me an iPhone XR if I get a hair straightener.
If you bring me a hair straightener then I don’t want a selfie stick. If you either give me a Netflix subscription or a pair of Adidas Yeezy Sneakers (but not both), then I’d like to receive tickets for Mark Forster if I don’t get the new Ed Sheeran album. If you grant my wish for a Netflix subscription, then,

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1 Wish list adapted from a list by Tobias Schubert and Sabrina Reinshagen.
2 https://rise4fun.com/Z3
if I don’t get an *iPhone XR* but I do get *tickets for Mark Forster*, I don’t want a selfie stick. And if you are not going to give me the *sixth A Song of Ice and Fire book*, then please bring a *hair straightener* to my birthday party.

And these are all of my wishes! See you at the party!

Yours, Annika

You may submit an SMT script via email or the formula together with a satisfying assignment written down on paper.