



Tutorial for Program Verification Exercise Sheet 4

In this exercise sheet we work with \mathcal{N}_{FOL} , the *Natural Deduction* proof system for first-order logic.

Submit your solution by uploading it as PDF in ILIAS.

Exercise 1: Natural Deduction Proofs for First-Order Logic 2 Points

Pick one of the following implications and use the Natural Deduction proof system for First-Order Logic \mathcal{N}_{FOL} to show that the implication holds.

- $\{(\forall x. p(x)) \vee (\forall x. q(x))\} \vDash \forall x. p(x) \vee q(x)$
- $\{\exists x. \forall y. p(x, y)\} \vDash \forall y. \exists x. p(x, y)$

If you solve this task for both implications you get additionally two bonus points.

Exercise 2: Proof rules of \mathcal{N}_{FOL} 2 Points

In the slides, you will find the rules for the Natural Deduction proof system for First-Order Logic \mathcal{N}_{FOL} . The proof system \mathcal{N}_{FOL} extends the proof system \mathcal{N}_{PL} by four rules.

Show that the side conditions of the rules (I_{\forall}) and (E_{\exists}) are necessary for the correctness of the rules. That is, for each of the rules, ignore the side condition and give an example such that the implication above the horizontal line holds, but the implication below the horizontal line does not hold.