In this exercise sheet, we work with the strongest postcondition $sp$ and its dual, the 
weakest precondition $wp$. These functions are also known as predicate transformers.

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Exercise 1: Strongest Postcondition for the Conditional Statement  2 Points
In the lecture we have seen that the strongest postcondition of a sequential composition 
or a while-loop can be expressed in terms of the strongest postconditions of the subst-
ations. In this exercise, you should give a similar formulation for the strongest 
postcondition of an conditional statement.

Specifically, let $st$ be the statement $\text{if (expr) \{ st_1 \} else \{ st_2 \}}$. Let $S \subseteq S_{V_{\mu}}$ be a 
set of states. Express the strongest postcondition $sp(S, st)$ using only the strongest 
postcondition of $st_1$, $st_2$ and of simple statements (havoc, assignments or $\text{assume}$) for 
suitable sets of states.

You do not have to prove the correctness of your result.

Exercise 2: Distributivity of $sp$  4 Points
In this exercise we examine distributivity properties of the strongest postcondition. Let 
$S, S_1, S_2$ be arbitrary sets of states, and let $st$ be a statement. Furthermore, let $\varphi_1$ and 
$\varphi_2$ be formulas.

For each of the following equalities, either prove its correctness or give a counterexample.

(a) $sp(S_1 \cup S_2, st) = sp(S_1, st) \cup sp(S_2, st)$
(b) $sp(S_1 \cap S_2, st) = sp(S_1, st) \cap sp(S_2, st)$
(c) $sp(S, \text{assume } \varphi_1 \lor \varphi_2) = sp(S, \text{assume } \varphi_1) \cup sp(S, \text{assume } \varphi_2)$
(d) $sp(S, \text{assume } \varphi_1 \land \varphi_2) = sp(S, \text{assume } \varphi_1) \cap sp(S, \text{assume } \varphi_2)$

Exercise 3: Strongest Postcondition  2 Points
Consider the following program $P$.

```
1 assume x > y;
2 x := x - y;
3 havoc z;
4 assume z > 0;
5 x := x * z;
```

Compute the strongest postcondition $sp(S, P)$ where $S$ is $\{ y > 0 \}$.
Exercise 4: Weakest Precondition

Analogously to the strongest postcondition we define the weakest precondition for a given set of states and a given statement $st$ as follows.

$$wp(S, st) = \{ s \in S_{\forall \mu} \mid \text{forall } s' \in S_{\forall \mu} \ (s, s') \in [st] \implies s' \in S \}$$

Intuitively, the weakest precondition is the set of states such that if we can execute $st$ and $st$ terminates then we are in some state of $S$.

Let us assume that the set $S$ is given by a formula $\psi$, i.e., $S = \{ \psi \}$. Give a formula $\varphi$ such that $wp(S, st) = \{ \varphi \}$ for the cases where

(a) $st$ is an assignment statement of the form $x := \text{expr}$,
(b) $st$ is an assume statement of the form $\text{assume expr}$, and
(c) $st$ is a havoc statement of the form $\text{havoc } x$. 
