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Hand in until 10:00 on April 26, 2021
Discussion: April 26, 2021

Tutorial for Program Verification
Exercise Sheet 1

Throughout the semester, we will publish exercise sheets every Monday (short exercise sheet) and every Wednesday (longer exercise sheet). Please complete the exercises, and upload a PDF of your solutions to ILIAS at the following URL:

`https://ilias.uni-freiburg.de/goto.php?target=crs_2134174`

In this exercise sheet we practice some of the key concepts of *Propositional Logic* (PL).

Exercise 1: Satisfiability, Validity 2 Points

Is the following PL formula satisfiable? Is the following PL formula valid?
If the formula is satisfiable then give a variable assignment such that formula is evaluated to **true**. If the formula is not valid then give a variable assignment such that formula is evaluated to **false**.

$$C \rightarrow (A \vee (B \wedge C))$$

Exercise 2: Conjunctive Normal Form 3 Points

We call two PL formulas F_1 and F_2 *logically equivalent*, denoted $F_1 \equiv F_2$, if they evaluate to the same truth value under every variable assignment.
We say that a formula F is in *conjunctive normal form* (CNF) if it is a conjunction of disjunctions of literals, i.e., if it has the form

$$\bigwedge_i \bigvee_j \ell_{ij}$$

where ℓ_{ij} are literals.
Any formula can be transformed into an equivalent formula in CNF using the following template equivalences (left to right):

$$\neg\neg F_1 \equiv F_1 \qquad \neg\mathbf{true} \equiv \mathbf{false} \qquad \neg\mathbf{false} \equiv \mathbf{true}$$

$$\left.\begin{array}{l} \neg(F_1 \wedge F_2) \equiv \neg F_1 \vee \neg F_2 \\ \neg(F_1 \vee F_2) \equiv \neg F_1 \wedge \neg F_2 \end{array}\right\}\text{De Morgan's Law}$$

$$F_1 \rightarrow F_2 \equiv \neg F_1 \vee F_2$$

$$F_1 \leftrightarrow F_2 \equiv (F_1 \rightarrow F_2) \wedge (F_2 \rightarrow F_1)$$
$$(F_1 \wedge F_2) \vee F_3 \equiv (F_1 \vee F_3) \wedge (F_2 \vee F_3)$$
$$F_1 \vee (F_2 \wedge F_3) \equiv (F_1 \vee F_2) \wedge (F_1 \vee F_3)$$

Transform the following formulas into an equivalent formula in CNF.

(a) $A \wedge B \rightarrow A \vee B$

(b) $C \rightarrow (A \vee (B \wedge C))$

Exercise 3: The NOR Connective 3 Points

In the lecture we defined the syntax of propositional logic by using only **false** and the logical connectives $\neg$ and $\wedge$. (The other logical connectives were introduced as abbreviations.) In this exercise we show that alternatively we could have defined the syntax of propositional logic by using only a single logical connective.

Given two PL formulas F_1 and F_2, we define the logical connective NOR ($\bar{\vee}$) by the following truth table:

F_1	F_2	$F_1 \bar{\vee} F2$
0	0	1
0	1	0
1	0	0
1	1	0

Table 1: Truth table for NOR

Show that the atom **false** and the logical connectives $\neg$ and $\wedge$ can be expressed by the NOR connective $\bar{\vee}$, i.e., given arbitrary PL formulas F, F_1, F_2 state for each of the PL formulas **false**, $\neg F$, and $(F_1 \wedge F_2)$ a PL formula that is logically equivalent but uses only F, F_1, F_2, and $\bar{\vee}$.

In this exercise it is sufficient to state a formula without a proof of logical equivalence.

Exercise 4: Birthday Wishes 4 Points

Annika was always a little excentric, but when she presented her family and friends with this year's list of birthday wishes (copied below)[1], they couldn't believe their eyes. Can you help them? Encode the constraints in boolean formulae and find a satisfying assignment in order to find a combination of presents that satisfies Annika's demands. You may use an SMT solver (e.g. Z3[2]) to obtain the satisfying assignment.

> To all my friends and family!
>
> you asked me what I wished for on my birthday, so here's my list:
>
> If one of my presents is going to be a *Netflix subscription*, then I don't want to receive the new *Ed Sheeran album*. If you are going to give me an *iPhone XR*, then I don't want a pair of *Adidas Yeezy Sneakers*. However, if you give me the *sixth A Song of Ice and Fire book*, then I would like the *Netflix subscription* and *tickets for Mark Forster*.
>
> If you do not get me *Adidas Yeezy Sneakers* as a present, then I want to receive the *sixth A Song of Ice and Fire book* or a *selfie stick*. If you do not bring me a *selfie stick*, then I ask you to bring me an *iPhone XR* if I get a *hair straightener*.
>
> If you bring me a *hair straightener* then I don't want a *selfie stick*. If you either give me a *Netflix subscription* or a pair of *Adidas Yeezy Sneakers* (but not both), then I'd like to receive *tickets for Mark Forster* if I don't get the new *Ed Sheeran album*. If you grant my wish for a *Netflix subscription*, then,

[1] Wish list adapted from a list by Tobias Schubert and Sabrina Reinshagen.

[2] https://rise4fun.com/Z3

if I don't get an *iPhone XR* but I do get *tickets for Mark Forster*, I don't want a selfie stick. And if you are not going to give me the *sixth A Song of Ice and Fire book*, then please bring a *hair straightener* to my birthday party.

And these are all of my wishes! See you at the party!

Yours, Annika

You may submit an SMT script, or the formula together with a satisfying assignment written down on paper.

Exercise 5: Natural Deduction Proofs 4 Points

Prove the following implications in the Natural Deduction proof system $\mathcal{N}_{\mathsf{PL}}$. That is, for an implication $\{F_1, \ldots, F_n\} \models F$, use the rules of $\mathcal{N}_{\mathsf{PL}}$ to build a derivation that shows this implication holds.

(a) $\{A \to B\} \models \neg B \to \neg A$

(b) $\{A \to (B \to C)\} \models A \land B \to C$