



Tutorial for Program Verification Exercise Sheet 4

In this exercise sheet we work with first-order theories,
in particular with *Peano Arithmetic*.

Submit your solution by uploading it as PDF in ILIAS.

Exercise 1: Sudoku in First-Order Logic

3 Points

Formalize the rules of Sudoku in First-Order Logic.

An n -Sudoku (for $n \in \mathbb{N}$) is given as $(n^2 \times n^2)$ -grid of numbers from 1 to n^2 . The grid is further divided into n^2 squares of size $(n \times n)$. Use the following predicate and function symbols:

- the binary equality predicate $\cdot = \cdot$,
- the binary function $num(\cdot, \cdot)$ such that $num(x, y)$ returns the number in column x and row y ,
- the binary function $square(\cdot, \cdot)$ such that $square(x, y)$ returns the the number of the square containing cell (x, y) .

Assume the underlying domain contains only the elements $\{1, \dots, n^2\}$ and the meaning of the equality predicate is already defined (e.g., because we consider the theory of equality).

- In every row, each number occurs at least once.
- In every row, each number occurs at most once.
- In every column, each number occurs at least once.
- In every column, each number occurs at most once.
- In every square, each number occurs at least once.
- In every square, each number occurs at most once.