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Hand in until 10:00 on July 05, 2021
Discussion: July 05, 2021

Tutorial for Program Verification
Exercise Sheet 19

In this exercise sheet we work with strongest postconditions, weakest preconditions; with abstract reachability graphs, and with programs with assert statements.

Submit your solution by uploading it as PDF in ILIAS.

Exercise 1: Strongest Postcondition and Weakest Precondition 3 Points

Let S and S' be sets of states, and let st be a statement. For each of the following set relations, either prove its correctness or give a counterexample.

(a) $S = wp(S', st) \quad \Leftrightarrow \quad sp(S, st) = S'$

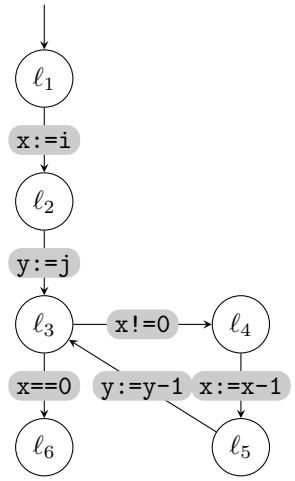
(b) $S \subseteq wp(S', st) \quad \Leftrightarrow \quad sp(S, st) \subseteq S'$

(c) $S \supseteq wp(S', st) \quad \Leftrightarrow \quad sp(S, st) \supseteq S'$

Exercise 2: Abstract Reachability Graph 4 Points

Consider the following Boo program P, with precondition $i = j$ and postcondition $x = y$.

```
1 x := i;
2 y := j;
3 while (x != 0) {
4   x := x - 1;
5   y := y - 1;
6 }
```



(a) Draw an abstract reachability graph for P that is precise for the set of formulas $B = \{i = j,\ i \neq j,\ x = i,\ y = j\}$.

(b) Give a set of formulas B' that is suitable to show correctness of the program, i.e., give a set B' and an abstract reachability graph (AC, T) for P that is precise for B', such that AC contains no configuration $(\ell_6, \{\varphi\})$ with $\{\varphi\} \cap \{\neg(x = y)\} \neq \emptyset$.

Exercise 3: Correctness Definitions 2 Points

In the lecture, we have seen how we can specify correctness of a program in terms of precondition-postcondition pairs or in terms of `assert` statements. In this exercise we will see how to relate the two concepts.

Given a program $P = (V, \mu, \mathcal{T})$ that contains an arbitrary number of `assert` statements, give a construction of a program P' and a precondition-postcondition pair $(\varphi_{\mathsf{pre}}, \varphi_{\mathsf{post}})$ such that the following holds:

P satisfies all `assert` statements
iff
P' satisfies the precondition-postcondition pair $(\varphi_{\mathsf{pre}}, \varphi_{\mathsf{post}})$.

Hint: *Introduce one or more new program variables. Make sure your transformation works for all programs, including the following two example programs:*

```
1 assert false;
2 while (true) {
3     x := x + 1;
4 }
```

```
1 x := 0;
2 assert x > 0;
3 assume x == 1;
```